ON PROPAGATION OF EXPLOSION AND IMPLOSION WAVES IN STELLER INTERIORS

THESIS PRESENTED
BY

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M.Sc., M. Phil.

FOR

THE DEGREE OF DOCTOR OF PHILOSOPHY (MATHS)

BUNDELKHAND UNIVERSITY
JHANSI (INDIA)

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1995

ACKNOWLEDGEMENT

I take this opportunity to express my deep sence of gratilude to Dr.V.K.Singh ,M.sc,Ph.D. A.P. (Reader), Department of Applied Mathmetics ,Institute of Engineering and Technology sitapur Road, Lucknow,for his guidance Valuable suggestions and keen interest throughout the progress of my research work.

It is highly desirable to express my indebtedness to Dr.P.N.Srivastava , Reader Head of the Maths & Stat . department, Bundelkhand University Jhansi,who had very kindly encouraged and Created inspirations in the field of Mathematics and its research work .

I am also thankful to Dr.H.C.Agarwal, Reader ,Head of the Maths Department , Bundelkhand College,Jhansi,for his precious comments and useful suggestions.

I am very much indebted to Dr. J. P. Agnihotri, Principal ,Bundelkhand College,Jhansi,for extending all the facilities during the period of study. I am thankfull to Mr. M.K. Gupta, my collegeage,now Lecturer, Meerut University Meerut. Dr.A.K.Agarwal and Yadvendra Shrivastava and E.C.A.Jhansi,for their valuable help

In the last but not least I feel highly indebted to my parents for their constant moral support, and I am also thankful to my wife who has been helpful in the most ardous hour of studies.

Date:-01/07/1995

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PREFACE

The present thesis an out come of researches carried out by me in the field Of "ON PROPAGATION OF EXPLOSIOH AND IMPLOSION WAVES IN STELLER INTERIORS" under the supervision of Dr.V.K.Singh M.sc. Ph.D , A.P. (Reader) in the department of Applied Mathematics Institute of Engineering and Technology Sitapur Road Lucknow , is being submitted for the award of Ph.D. degre in Mathematics . The thesis has been devided in to seven Chapter, each Chapter has futher sub devided into a number sections and subsections.The first Chapter is introduction illustrsting the basic equation involved in the out come of the present thesis . It briefly discussed the basic idea of newtonian fluid Law of Eulerian motion governing the flow, equation. Of laws of conservation of mass and the equation of energyinvolved therein .It also highlight how the discontinuities occur in case of sudden explosion the fundamentalequation & which governs the flow of the fluid particles behind the blast wave and corresponding jump in the physical variable has also be described . When such discontinuities pass through a conducting rigion the fundamental equation are coupled with the maxwell's electromegnetic equation and in the radiation phenomina due to ultra violet rays and X-Rays becomes important and the thermodynamic Laws play a significant role . A model of such equation have been derived in the subsequent section.

Chapter II models the propagations of spherical high temperature discontinuities ~~them~~ in a ionized atmosphere . A similarties solutions of the fundmental equation governing

these discontinuities due to sudden point explosion has been soughtunder the initial condition that the velocities of fluid partical tends to infinity in the region bounded by the explosion wave and else where the fluids particle are at rest.

In the Chapter III the idea Concieve from the Chapter IIhas been extanded in a self gravitating system in the interiors of star . The viscosity and heat Conduction effect are neglected for convenience and the entropy of the system is taken as constant along a straight line.

In the Chapter IV the propagation of the spherical exploding shock waves produced on account of sudden energy released in an ordinary gases of variable density has been discussed, the singularity points in the course of Integration has also been formulated .

The Chapter V discusses the growth and decay of the discontinuities in the case of thermal medium,the higher order compatible condition have been obtained .In application of the compatibity condition has been laid down in the subsequent sections.

In the Chapter VI the conductivity effect on an expanding pressure shock has been considered . The thickness of the such a shock is considered to be finite.The growth equations for such is shock has been derivied in thermally & electrically conducting [illegible] gas with radiation effect .

In last Chapter of the present out come , the Chapter VII,we discussed the differential effect of Isothermal shock where the heat addition in a electhrically conduciting gas is possible some particulur cases have also been discussed across the shock surface.

CONTENTS

CHAPTER - I

Introduction

Newton's principle:-

The theory of fluid flow(for an incompressible fluid whether liquid or gas is bases on the Newtonian fluids.The essential part of the Newton's principle can be formulated into the folowing statments :

(a) To each particle of the fluids can be assigned a positive number m in variant in time and called its mass;and

(b) The particle moves in such a way that at each moment the product of its acceleration vector by m is equal to the sum of certain other vectors called forces which are determined by the Circumstances under which the motion take place (Newton's second Law).

By means of a limiting process,this principle can be adapted to the case of continuum in which a velocity vector $\vec{U}$ and an acceleration vector $d\vec{U}/dt$ exist at each point with co-ordinates (x,y,z) or position $\vec{r}$ and dv a volume element in the neighbourhood of p , to this volume element will be assigned a mass ρ dv where ρ is the density or mass per unit volume . Density are measured in Kg/m^3 . The forces acting upon this element are,the external force of gravity ρ g dv and the internal forces resulting from interaction with adjecent volume element. Thus, after deviding by dv the relation

$$\rho \frac{d\vec{U}}{dt} = \rho g + \text{internal force per unit volume} \quad (1.1)$$

is a first expression of statement (b)

To formulate part (a) of Newton's principle note that the mass to be assigned to any finite portion of the continuum is given by $\int \rho dv$ and therefore since this is invariant with respect to time C.F.[1].

$$\frac{d}{dt} \int \rho dv = o \quad (1.2)$$

First the meaning of the differentiation symbol d/dt occuring in equation [1.1] and [1.2] are as per convention.

The density P and velcity vector $\vec{u}$ are each considered as function of the four variable of space and time such as (x,y,z) and t, so that partial derivatives with respect to time and with respect to the space co-ordinate may be taken as well as the direction l is given by

$$\frac{\partial}{\partial t} = \cos(l,x) \frac{\partial}{\partial x} + \cos(l,y) \frac{\partial}{\partial y} + \cos(l.z) \frac{\partial}{\partial z}$$

where cos (l,x),cos (l,y),and cos (l,z)

Are the direction cosine defining the directionl.In particular;if l be taken as the direftion of $\vec{U}$ the dlirection cosine may be expressed in term of $\vec{U}$ to give

$$\vec{U}\frac{\partial}{\partial s} = \vec{U}_x \frac{\partial}{\partial x} + \vec{U}_y \frac{\partial}{\partial y} + \vec{U}_z \frac{\partial}{\partial z} \qquad (1.3)$$

where s is used in place of 1 to designate the direction of the line of flow; for this direction

$ds = \vec{U}\ dt$. By d/dt in equations (1.1) and (1.2) is meant not partial differentiation with respect to t at constant (x,y,z) but rather differntiation for a given particle where position changes according to equation (1.3)

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \frac{dx}{dt}\frac{\partial}{\partial x} + \frac{dy}{dt}\frac{\partial}{\partial y} + \frac{dz}{dt}\frac{\partial}{\partial z}$$

$$= \frac{\partial}{\partial t} + \vec{U}_x \frac{\partial}{\partial x} + \vec{U}_y \frac{\partial}{\partial y} + \vec{U}_z \frac{\partial}{\partial z} = \frac{\partial}{\partial t} + \vec{U}\frac{\partial}{\partial s} \qquad (1.4)$$

the acceleration Vector $d\vec{U}/dt$ is the time rate of change of the velocity vector $\vec{U}$ for a definite material particle which moves in the direction of $\vec{U}$ at the rate $\vec{U} = ds/dt$ the operation d/dt may be termed particle differntiation or material differentiation with respect to time at a fixed position . An alterna tive for of equation (1.4) is

$$\frac{d}{dt} = \frac{\partial}{\partial t} + (\vec{U}\ .\ \text{grad}) \qquad (1.5)$$

ere grad is considered as a symbolic vector with the component

$\partial/\partial x$, $\partial/\partial y$ and $\partial/\partial z$ in accordance with the well known notation of grad for the vector with components $\partial f/\partial x$, $\partial f/\partial y$ and $\partial f/\partial z$, which is called the gradient of f and the scalar product $\vec{U}\cdot$ grad means the product U times the component of grad in the U direction i-e

$$\vec{U}\frac{\partial}{\partial s}=\vec{U}_x\frac{\partial}{\partial x}+\vec{U}_y\frac{\partial}{\partial y}+\vec{U}_z\frac{\partial}{\partial z}$$

equation (1.4) or (1.5) will be represent to as the Euler rule of differentiation

2.Newton's Law Of Motion For An Inviscid Fluid

The equation (1.1) holds for any continuously distributed mass in which the density ρ is defined at each point and at each moment of time . By inviscid fluid is meant that the force acting on any surface element .ds at which two element of the fluid are in contact acts in direction normal to the surface element.At each point P the stress or the force per unit area is independent of the orientation (direction of the normal) of ds . The value of stress is called the hydraulic pressure or briefly pressure, p,at the point P for a small rectangular cell of fluid of V volume dv = dx dy dz two pressure forces act on the x-direction on surface elements of area dy dz. Taking the x-axis toward the right the left hand face experiences a force p dy dz directed toward the right which the right- hand face experiences a force (p+dp) dy dz directed toward the left here dp =($\partial p/\partial x$)dx . The resultant force s in the x-direction is thus

$$dp\, dy\, dz = -\frac{\partial p}{\partial x} dx\, dy\, dz = -\frac{\partial p}{\partial x} dv$$

Therefore the internal force per unit volume appearing in equation (1.1) has x-component $-(\partial p/\partial x)$

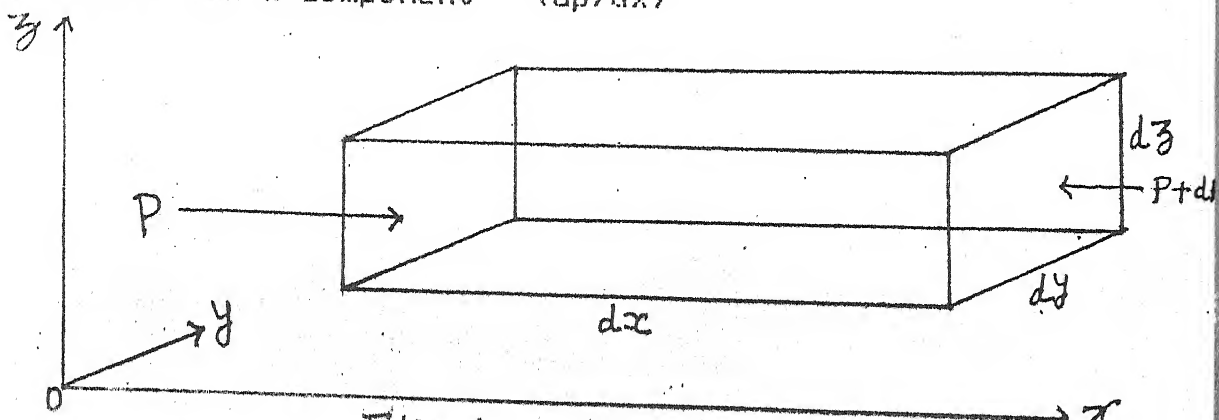

Fig- 1

Similarly the remaining component are found to be $-(\partial p/\partial y)$ and $-(\partial p/\partial z)$ Hence,

$$\rho \frac{d\vec{U}}{dt} = \rho\, g - (\mathrm{grad}\ p) \qquad (1.6)$$

expresses part (b) of Newton's princip le . The equation was first given by Euler [1]. but is usually reffered as Newton's equation of motion.

The vector equation (1.6) is equivalent to the three scalar equations along the rectangular axes (x,y,z)

$$\rho \frac{d}{dt} \vec{U}_x = \rho\, g_x - \frac{\partial p}{\partial x}$$

$$\rho \frac{d}{dt} \vec{U}_y = \rho\, g_y - \frac{\partial p}{\partial y} \qquad (1.7)$$

$$\rho \frac{\partial}{\partial t} \vec{U}_z = \rho g_z - \frac{\partial p}{\partial z}$$

equation (1.6) and (1.7) are valid only for inviscid fluid particles . If viscosity is present additional terms must be included in the expression for the internal force per unit volume C.F. [2]

3.EQUATION OF CONTINUITY

In order to express part a of Newton's principle conservation of mass in the form of differential equation the differentition indicated in equation (1.2) could be carried out by trans for ming the integral suitably . It is simpler however to consider the rectanguir cell or fig.1 fluid mass of flow into the cell through the left hand face at the rate of ρU_x dy dz units of mass per second and out of the right-hand face at rate $[\rho\vec{U}_x + d(\rho\vec{U}_x)]$ dy dz, where $d(\rho\vec{U}_x)$ is the product of dx and rate of change of $\rho \vec{U}_x$ in the x-direction,or $[\partial/\partial x(\rho\vec{U}_x)]$ dx . Thus the net increase in the amount of mass present in this volume element caused by flow across these two faces is given by $-[\partial/\partial x(\rho\vec{U}_x)]$ dx dy dz with analogous expressions for the other pairs of faces. If we use the expression div V divergence of the vector v for the sum

$$\left(\frac{\partial}{\partial x} V_x\right) + \left(\frac{\partial}{\partial y} V_y\right) + \left(\frac{\partial}{\partial z} V_z\right)$$ the total change

in mass per unit time is $-$ div $[(\rho\vec{U})dV]$. Now if the mass of each moving particle is invariant in time . According to law of

conservation of mass difference between the mass entering the cell and that leaving the cell must be balanced by a change in the density of the mass present in the cell . At first the mass in the cell is given by ρdV and after time dt by $(\rho+d\rho)\,dV$ where $d\rho = (\partial\rho/\partial t)dt$. Thus the rate of change of mass in the cell, per unit tim is given by $(\partial\rho/\partial t)\,dt$ so that

$$\operatorname{div}(\rho\vec{U}) = -\frac{d\rho}{dt} \qquad (1.8)$$

This relation valid for any type of continuously distributed mass is known as the equation of continuity .

A slightly different from of (1.4) is obtained by carring out the differentiation of ρU giving C.F. [1]

$$U_x\frac{\partial\rho}{\partial x} + U_y\frac{\partial\rho}{\partial y} + U_z\frac{\partial\rho}{\partial z} + \rho \operatorname{div}\vec{U} = -\frac{\partial\rho}{\partial t}$$

or , using Euler rule,

$$\frac{d\rho}{dt} + \rho \operatorname{div}\vec{U} = 0 \qquad (1.9)$$

EQUATION OF STATE

The most common form of specifying equation consists in the assumption that P and ρ are variable but connected at all time by one to one relation of the form

$$F(p,\rho) = 0 \qquad (1.10)$$

This means that if the pressure is the same at any two point the density is also the same at these two point whether at the same or different moments in time.

Examples of such (p,ρ) relation are

$$\frac{p}{\rho} = \text{Constant} \tag{1.11}$$

$$\frac{p}{\rho^{K}} = \text{Constant} \tag{1.12}$$

$$p = A - \frac{B}{\rho} \quad , \quad B > o \tag{1.13}$$

Where K, A and B are constant

In the first of example (1.11–1.13) pressure density are proportional in genral it will be assumed that ρ increases at p increases, and vice versa so that $dp/d\rho > o$. If the specfying equation is of the form (1.10) , the fluad is called an elastic fluid because of the analogy to the case of an elastic solid where the state of stress and the state of strain determine each other A large part of the result s so far obtained in the theory of compressible fluid hold for elastic fluid only C.F. [1].The entropy s of a perfect gas is difined by

$$S = \frac{gR}{(\gamma-1)} \log \frac{p}{\rho^{\gamma}} + \text{constant} \tag{1.14}$$

Where γ is constant .

Thus the motion of a perfect gas with condition (1.12) as specifying equation and $k = \gamma$ is isentropic i.e

$$\left[\frac{p}{\rho^{\gamma}} \right] = \text{Constant.} \qquad (1.15)$$

The motion of any fluid having of specifying equation (1.11) with $k > 1$ will be termed polytropic.

5.EQUATION OF ENERGY

In the many cases the specification of the type of flow is given in the thermodynamic terms. It is then necessary in order to set the specifying equation (1.10) to expresses these thermodynamic variables . It is known (or assumed) that the temperature is equal at all points for all values of t then the equation of state which is relation between T, p and ρ supplies relation of the form (1.10)

$$F(p, \rho) = 0$$

The most common assumpition in the study of compressible fluid is that no heat out put or input occurs for any particle . If this refers to heat transfer by radiation and chemical process only the flow is called simply adiabatic . If heat conduction between neighbouring particle is also excluded we speak of stric tly adiabatic motion.

In order to translate either assumption into a specifying equation the first law of thermodynamics must be used which gives the relation between heat input and the mechanical variables . If Q'denotes the total heat input from all source per

unit of time and mass the first Law for an inviscid fluid can be written

$$Q' = C_v \frac{dT}{dt} + p \frac{d}{dt} \left(\frac{1}{\rho}\right) \qquad (1.16)$$

where C_v is the specific heat of the fluid at constant volume and , the quantity of heat is measured in mechanical units . The first term on the right represents the part of the heat input expended for the increase in temperature ;the second term corresponds to the work done by expansion .

Equation (1.16) is equalvalent to the more familiar equation

$$d_Q = c_v \, dT + p dv$$

which is derived from (1.16) by multiplying by dt and writting V (specific volume) in place of $(1/\rho)$

It is known that a flow is stricitly adiabatic i.e that the total heat input from any source (conducting radiation etc) in zero then Q'has to be set equal to zero in (1.16) while T,may be expressedin terms of p and ρ by means of equation of state.

Finally an expression for c_v in term of the variable T,p and ρ is needed for a perfect gas where the equation of state is (1.15) it is generally assumed that C_v is constant given by c.f (R)

$$C_v = \frac{gR}{\tau - 1} \qquad (1.18)$$

where for dry air τ = 1.4 we note for further reference that from equation (1.15)

$$C_v T = \frac{1}{\tau - 1} \frac{p}{\rho} \qquad (1.19)$$

hus from (1.15), (1.16) and (1.17)

$$\frac{1}{\gamma-1}\frac{p}{\rho}\frac{d}{dt}(\log\frac{p}{\rho^{\gamma}}) \qquad (1.20)$$

for a perfect inviscid gas. With the assumption that $Q' = o$ quation (1.20) reduces to the specifying equation

$$\frac{d}{dt}(\log\frac{p}{\rho}) = o \qquad (1.21)$$

lding for strictly adiabationc flow of perfect inviscid qas . means of (1.15) & (1.14) equation (1.20) and (1.21) may be pressed in terms of the entropy s, giving

$$Q' = T\frac{ds}{dt} \quad , \quad \frac{ds}{dT} = o \qquad (1.22)$$

us , S (or $\frac{p}{\rho^{\gamma}}$) keeps the same

lue for each particle at all time when $Q' = 0$ Nevertheless this nstant value of s may be different for distinct particle so at strictly adiabatic flow need not also be irentropic.

6. Origin of Strong Discontinuties

When a fluid moving with a supersonic velocity meets an obstacle , a type of wave known as shock wave is formed . It may result from violent disturbances due to various causes ,for example, detonation of explosives ,flow through rocket nozzles, supersonic flight of projectiles and so on. Shock waves are the most conspicuous phenomenon occuring in non-linear wave propagation. Even without being caused by initial discontinuities, they may appear and be propagated . The underlying mathematical fact is that unlike linear partial differential equations, non-linear equation often do not admit solutions which can be continously extended whenever the differential equations themselves remain regular. The concept of the shock waves goes almost a century back to Riemann , Who was the first to recognise the essential difference between the propagation of infinitely small and of finite pressure variations. The theory was later developed by Rankine, Hugoniot, Hadamard and many problem outside of supersonic aeronautics - for example, detonation waves , but also has great importance for several practical aeronautical problems . In fact shock waves may cause sudden change in the aerodyanmic behaviour of high speed aircraft affecting their balance , stability and control producing undesirable vibrations . The physical reason why a discontinuous change is possible only in a supersonic flow can be easily seen. The theorem of conservation of mass calls for the equality of the so called Fano number on both sides of the discontinuity surface. When we consider the expansion or compression process of a gas , we find that the Fano number has a

maximum when the velocity of the gas is equal to the velocity of sound . Consequently in the case of a blast wave on account of sudden explosion , the velocity normal to the discontinuity surface has to be subsonic on one side and supersonic on the other therefore, no discontinuous change can occur in a purely sobsonic steady flow, The conditions for the existence of a detonation wave as given by Hayes [9] are,

(i) The conservation laws must be satisfied . This condition will , however , be necessary but not sufficient.

(ii) The specific entropy of the material must increase.

(iii) The discontinuity must correspond in its structure to a physically realizable process. This condition is necessary and sufficient for the existence of the discontiunuity in the small provided the discontinuity is internally stable. Since the appropriate physical and thermodynamical laws must be satisifed within the structure of the discontinuity , conditions (i) and (ii) are automatically satisfied.

(iv) The discontinuity must be internally stable;

this means that if an equilibrium solution undergoes a disturbance allowed by the local hydrodynamic conditions, the solution must return to the equilibrium one.

7. Equations Governing the Detonating wave Propagation and Jump Conditions :-

As described earlier if ρ and P be the density and pressure at a point P , then in the continuous media the equations governing the flow are , [4], [5], [6],

$$\frac{\partial \rho}{\partial t} + u_i \rho_{,i} + \rho u_{i,i} = 0 \qquad 1.22$$

$$\rho \frac{\partial u_i}{\partial t} + \rho \; u_j \; u_{i,j} + p_{,i} = 0 \qquad 1.23$$

$$\rho \left(\frac{de}{dt} - \frac{p}{\rho^2} \frac{d\rho}{dt} \right) = 0 \qquad 1.24$$

$$\frac{p}{\rho} = g\,R\,T \qquad 1.25$$

Where summation convention has been adopted and a (,) comma followed by an index donotes partial derivative with respect to x_i, u is the particle velocity given by

$$U^2 = U_i \; U_i \qquad 1.26$$

and e is the internal energy per unit mass .. Also, as usual

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + U_i \, f_{,i} \qquad 1.27$$

is the total derivative following the fluid motion . The equation representing the conservations of mass momentum and energy across the surface of discontinuities (due to sudden explosion) are,

$$\rho_1 \; (U_{1i} - G) \; n_i = \rho_2 \; (U_{2i} - G) \; n_i = m \qquad 1.28$$

$$[\, p \,] \; n_i = -m \; [\, U_i \,] \qquad 1.29$$

$$\frac{m}{2} \; (U_1^2 - U_2^2) - m[I] = p_1 \, U_{1i} \, n_i - p_2 \, U_{2i} \, n_i \qquad 1.30$$

Where the subscripts 1 and 2 denote the corresponding quantity infront (region 1) of and behind (region 2) the Blast surface , n_i are the components of the unit normal to the discontinuity directed from the region 1 to the region 2 , G is the velocity of the associated shock and the bracket [ ] denotes the difference of values on the two sides of the associated shock surface of the quantity enclosed . The equation (1.28–1.30) are the so called Rankine –Hugoniot equation c.f Taylor and Maccoll [7]

The expressions for the flow quantities behind the shock surface in term of thes quantities infront of the shok are.

$$[U_i] = -\frac{\delta}{(1+\delta)} U_{1n} n_i \tag{1.31}$$

and

$$[p] = \frac{\delta}{(1+\delta)} \rho_1 U_{1n}^2 \tag{1.32}$$

where δ is the shock strength defind by Truesdell and is given by

$$\delta = \frac{[\rho]}{\rho_1} \tag{1.33}$$

8. Flow and field equation in a conducting region

The interaction between hydrodynamic motion and magnetic field in a conducting gas is of importance in problems of Astrophysics ,Geophysics and the behaviour of interstellar gas masses. In a super conductive gas region,we study the motion of

an electrically conducting fluid in the presence of a magnetic field . On account of the motion of the gas , electric currents are induced and modify the flow . The interest and difficulty of this interaction between the field and the flow from the subject matter of magnetogasdynamics . As is done in many practical problems,we have through out the thesis ignored the Maxwell's displacement currents and considered the motion of a continuous conducting gas . Also the magnetic permeability (μ) has been mostly taken as unity. The field equations are,[9],[10]

$$\operatorname{div} \underline{H} = o \tag{1.34}$$

$$\text{Curl } \underline{H} = \frac{4\pi \underline{J}}{e} \tag{1.35}$$

$$\operatorname{div} \underline{J} = o \tag{1.36}$$

$$\text{Curl } \underline{E} = - \frac{1}{e} \frac{d\underline{H}}{dt} \tag{1.37}$$

where the symbple $\underline{H}, \underline{J}, \underline{E}$ and c denote the magnetic field current density ,electric intensity and velocity of light respectively.

If the fluid has velocity U ,the electric field which it experinces is $\underline{E} + \underline{U} \times \frac{\underline{H}}{c}$, Thus if σ is the electrical conductivity,then

$$\underline{J} = \sigma \left(\underline{E} + \frac{\underline{U} \times \underline{H}}{e} \right) \tag{1.38}$$

As a consequence of (1.35) and (1.38) we have

$$\operatorname{curl} \underline{H} = \frac{4\pi\sigma}{c} \left(\underline{E} + \frac{\underline{U} \times \underline{H}}{c} \right) \qquad (1.39)$$

If we express $\underline{E}$ in terms of $\underline{H}$ under certain magnetogasdynamic approximations [11] and substitute in the equation (1.37)

$$\frac{\partial H}{\partial t} = \operatorname{Curl} \left(\underline{U} \times \underline{H} \right) + \left(\frac{c^2}{4\pi\sigma} \right) \Delta \underline{H} \qquad (1.40)$$

If we assume ,as usual, that the dissipative mechanisms, such as viscosity ,thermal conductivity and electrical resistance are absent , the equations governing the coupled motion of magnetogasdynamic s are [12],

$$\frac{\partial H_i}{\partial t} - U_{i,j} H_j + H_{i,j} U_j + H_i U_{k,k} = 0 \qquad (1.41)$$

$$H_{i,i} = 0 \qquad (1.42)$$

$$\rho \frac{dU_i}{dt} + p_{,i} + \frac{1}{4\pi} H_k H_{k,i} - \frac{1}{4\pi} H_j H_{i,j} = 0 \qquad (1.43)$$

$$\frac{dp}{dt} + \rho U_{i,j} = 0 \qquad (1.44)$$

$$\rho \frac{dh}{dt} = \frac{dp}{dt} \qquad (1.45)$$

where h is the sepecific enthalpy defined as

$$h = e + \frac{p}{\rho} \qquad (1.45)$$

On the basis of the laws of conservation of mass, momentum and energy and Maxwell's electromagentic equation, several types of discontinuities can exist in ideal electricaly conducting fluids in the presence of magnetic fields [13]. The discontinuties characterized by the condition that both the mass flow and density change across them are different from zero are called shock waves. The study of magnetohydrodynamic shock waves was begun in 1950 with the paper of F. de Hoffmann and Teller [14]. Since then continued interest inspired by astrophysics , by the possibilities of thermonuclear power , by fligh at the outer edges of the atmosphere , etc., has prouduced many papers describing shock wave properties . The basic properties of magnetohydrodynamic blast waves as determined by the conservation laws (the Rankine-Hugoniot relations) have been deveploped further by Friedrichs [15] Helfer [16], Lust [17], Bazer & Ericksons [18], Kanwal [12] and many others, and they are now well understood but the more complex question of their existence in nature has yet to be exhaustively treated inspite of efforts in this direction by several Russian authors, In the presence of a magnetic field the relation connecting the flow and field quantities on the two sides of the associated shock surface are [12], [25], [26] :

$$H_{e1}\,U_{en} - H_{in}\,V_{in} = H_{in}\,[V_i] \quad (1.47)$$

$$H_{en} = H_{in} \quad (1.48)$$

$$[V_i]\rho_i V_{in} = -\left[p+\frac{H^2}{8\pi}\right] n_i + \frac{1}{4\pi} H_{in}[H_i] \quad [1.49]$$

$$\rho_i V_{in} = \rho_2 V_{en} \quad (1.50)$$

$$\left[\rho\left(\frac{V^2}{2} + h + \frac{H^2}{4\pi\rho}\right)V_n\right] - [H_i V_i]\frac{H_{in}}{4\pi} = 0 \quad (1.51)$$

where

$$[V_i] = [U_i - G_{ni}] = [U_i] \quad (1.52)$$

$$H_n = H_i n_i \quad (1.53)$$

$$V_n = V_i n_i \quad (1.54)$$

The expressions for the flow and field quantities behind the associated shock surface on account of sudden explostion in terms of these quantities infront of the shock surface are [12]

$$[H_i] = \frac{4\pi\delta\rho_i V_{in}^2 H_{i\beta} X_{i,\beta}}{(4\pi\rho_i V_{in}^2 - (1+\delta)H_{in}^2)} \quad (1.55)$$

$$[V_i] = -\frac{\delta}{(1+\delta)} V_{in}\, n_i + \frac{\delta V_{in}\, H_i\, \alpha\, H_{in}\, X_{i,\alpha}}{(4\pi\rho_i V_{in}^2 - (1+\delta)H_{in}^2)} \quad 1.56$$

$$[p] = \frac{\delta}{(1+\delta)} \rho_1 v_{1n}^2 - \left(\frac{\delta \rho_1 v_{1n}^2}{(4\pi\rho_1 v_{1n}^2 - (1+\delta)H_{1n}^2)} + \frac{2\pi\delta^2 \rho_1^2 v_{1n}^4}{(4\pi \rho_1 v_{1n}^2 - (1+\delta)H_{1n}^2)} \right) H_{\tau 1}^2 \quad 1.57$$

where

$$H\alpha = H_i X_{i,\alpha} \quad (1.58)$$

9. Effect of Radiation

Strong blast waves matter to high temperatures and when the temperatures are high ,or X-ray range,of the spectrum, depending upon the temperature . For this reason , radiation play an impròtant role in many hydrodynamic processes relevant to strong blast waves and explosions .Often the role of radiation is not confined to luminescence of the heated body . It can take part in the energy transfer and heat exchange and cause energy losses ,thus affecting the hydrodynamic movement of matter . At very high temperature if the medium is rarefied but extended the energy and pressure of radiation become comparable with those of matter , and therefore, influnce the thermodynamic properties of the medium.[19]

Radiation phenomena have acquired an interest for gas dynamics mainly since attention has been attracted in science and technology to such phenomena as nuclear explosions , hypersonic motion of bodies in the atmosphere, powerful electric discharge and astronomical problems.Quite recently have been found , namely

those connected with the interaction of intensive laser emission with matter [20].

THERMODYNAMIC CONSIDERATIONS.

We now consider the application of thermodynamics to inclosures containing radiation . Consider a perfectly black body M, contained in an inclosure with perfectly reflecting walls. The inclosure will be traversed in all direction by radiation. Let thetemperature of the black body contained in the inclosure be T. In a steady state ,the inclosufe is traversed by "black body radiation" at temperature T.We assume that quasistatical processes can be carried out with the radiation and suppose , further that the radiation is the same throughout the inclosure . Let the energy of radiation per unit volume be E_R ,so that internal energy by .

$$e = E_R V \qquad (1.59)$$

There is certain analogy between radiation and a perfect gas . The energy of both depends on temperature and both exert pressure. According to the electromagnetic theory of light , radiation exerts the pressure,c.f.[21]

$$P_R = 1/3 \quad E_R \qquad (1.60)$$

Let us allow the inclosure to expand quasistatically while the temperature is maintained constant

Let the Volume , V , increase

by an amount dv while E_R and P_R remain unaltered .Consequantly ,the internal energy increases by an amount E_R so that

$$\left(\frac{\partial e}{\partial v}\right)_T = E_R \qquad (1.61)$$

We shall now use the thermodynamical formula [18]

$$\left(\frac{\partial e}{\partial v}\right)_T = T\left(\frac{\partial p_R}{\partial T}\right)_V - p_R \qquad (1.62)$$

The radiation pressure P_R is assumed to depend only on T, and then as a consequence of the equation (1.60) and (1.61) the equation (1.62) can be written as [22]

$$E_R = 1/3\, T\,\frac{dE_R}{dT} - 1/3\, E_R \qquad (1.63)$$

or

$$T\,\frac{dE_R}{dT} = 4E_R \qquad (1.64)$$

or, we may write,

$$E_R = aT^4 \;,\quad p_R = 1/3\, a\, T^4 \qquad (1.65)$$

Thus, the energy of black body radiation per unit volume is proportional to the fourth power of the temperature . this is known as Stefan's Law and the constant a is called Stefan-Boltzmann constant

9. HEAT FLUX OF RADIATION.

The net amount of energy passing thruugh the surface per unit time is called the flux through the surface and for optically thick medium it is given by the expression (23).

$$F_R = - D_R \text{ grad } E_R \qquad (1.66)$$

where

$$D_R = \frac{cl}{3} \qquad (1.67)$$

is the Rosseland diffusion coefficient of radiation and ,

$$l = \frac{1}{k\rho} \qquad (1.68)$$

l being the Rosseland meanfrace path of radiation , c is the velocity of light and k, which depends upon temperature T and density ρ, is the opacity or Rosseland mean absorption coefficient.

10.FUNDAMENTAL EQUATION AND JUMP CONDITIONS IN RADIATION GASDYNAMICS.

Neglecting the effect of viscosity and heat conductivity the fundamental equations in radiation gasdynamics are c.f.(13)&(24)

$$\frac{d\rho}{dt} + \rho\, U_{i,i} = 0 \qquad (1.69)$$

$$\rho \frac{dU_i}{dt} = - (p_m + p_R)_{,i} \qquad (1.70)$$

$$\rho \frac{d}{dt}\left(\epsilon + \frac{E_R}{\rho}\right) + (p_m + p_R)\, U_{i,i} + F_{R_{i,i}} = 0 \qquad 1.71$$

p_m being the material pressure.

We consider the existence of a stationary shock and assume , for simplicity that the motion is one dimensional so that the quantities are function of x only . The equations (1.69) to (1.71) can then be written as.

$$\frac{d}{dx}(\rho U) = o \qquad (1.72)$$

$$\rho U \frac{dU}{dx} = -\frac{d}{dx}\left(p_m + \frac{aT^4}{3}\right) \qquad (1.73)$$

$$\rho U \frac{d}{dx}\left(e + \frac{aT^4}{\rho}\right) = \left(p_m + \frac{aT^4}{3}\right)\frac{dU}{dx} - \frac{d}{dx}F_R \qquad (1.74)$$

Integrating (1.72) to (1.74) we get

$$\rho U = m \qquad (1.75)$$

$$p_m + \frac{aT^4}{3} + mU = C_1 \qquad (1.76)$$

$$m\left(e + \frac{aT^4}{\rho}\right) + \tfrac{1}{2}mU^2 + mc_1U = C_2 \qquad (1.77)$$

Equation (1.75) to (1.77) are called jump conditions across the shock with radiation . The Mach number of the shock may be defined as

$$m = \frac{U}{a_s} \tag{1.78}$$

where a_s is called as the speed of sound in the medium and is given [21] as,

$$a_s^2 = \frac{\partial\left(p_m + \frac{aT^4}{3}\right)}{\partial\rho} = \frac{p_m}{\rho}\left\{\frac{\tau+20(\tau-1)\eta+16(\tau-1)\eta^2}{1+12(\tau-1)\eta}\right\} \tag{1.79}$$

where "τ" is the usual adiabatic exponent and η is defined as

$$\eta = \frac{p_r}{p_m} \tag{1.80}$$

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THE PROPAGATION OF POINT EXPLOSION WAVE IN ATMOSPHERE AN IONIZED

1. INTRODUCTION.

Roger a chevelier [1] etal discussed linear analysis of oscillatory and taylor [2] discussed the solutions of the equation of motion produced by a strong point explotion causing the propagation of an associated spherical shock wave in an ordinary gas . Lin [3] obained solution in the case of a cylindrical shock wave produced on account of instantaneous energy release along an infinite straight line . Verma [4] obtained equations for the solution of the problems of a cylindrical blast wave in a conducting gas.following Verma.this part of the chapter is intended to discuss the solutions of the equation of motion in the case of a spherical blast wave produced by a sudden point explosion and propagating in a conducting gas otherwise at rest . The disturbance is bounded on the out side by a spherical shock that moves symmetrically outside. Viscosity and heat conduction are neglected and it is assumed that the flow is isentropic along a stream line.

2. FUNDAMENTAL EQUATION

The equation governing the flow and field are,

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right) + \frac{\partial p}{\partial x} + H\frac{\partial H}{\partial x} = 0 \qquad (2.1)$$

$$\frac{\partial \rho}{\partial t} + U \frac{\partial \rho}{\partial x} + \rho \left(\frac{\partial U}{\partial x} + \frac{2U}{x} \right) = 0 \qquad (2.2)$$

$$\frac{\partial H}{\partial t} + U \frac{\partial H}{\partial x} + H \left(\frac{\partial U}{\partial x} + \frac{2U}{x} \right) = 0 \qquad (2.3)$$

$$\left(\frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right) (p \rho^{-\gamma}) = 0 \qquad (2.4)$$

where U, H, P and ρ are respectively the velocity, the magnetic field strength pressure and density of the flow at a radial distance x from the point of explosion at any time t. The medium is assumed to be a perfect gas so that $p = a \rho^{\gamma}$. The motion is bounded on the outside by the shock surface, x = R (t) which moves outward with a velocity say U = dR/dt. We assume that ahead of the shock the undisturbed pressure density and magnetic field are p_0, ρ_0 and H_0 respectively.

3. NON DIMENSIONAL SIMILARITY TRANSFORMATIONS

The solutions may be sought by the following similarity assumptions by writting the unknows in the form as given in [5]

$$p = p_0 R^m f_1 (\eta) \qquad (2.5)$$

$$\rho = \rho_0 \psi_1 (\eta) \qquad (2.6)$$

$$U = R^n \phi_1 (\eta) \qquad (2.7)$$

$$H = H_0 R^k g_1 (\eta) \qquad (2.8)$$

where $\eta = \frac{x}{R}$ is a nondimensional radial variable and f_1, ϕ_1, ψ_1 and g_1 are function of η only . Substituting (2.5) to (2.8) in the equation (2.1) , we get

$$\frac{\dot{R}}{R^n}\left(\eta\,\phi_1 + n\phi_1'\right) + \phi_1\phi_1' = -\frac{1}{\rho_0\psi_1}$$

$$\{P_0 R^{m-2n}\, f_1' + H_0 R^{(2k-2n)}\, g_1'\} \qquad (2.9)$$

In order that all the unknowns may be expressible as functions of η alone , the following relations must be fulfilled.[6]

$$m = 2n \;,\; k = n \qquad (2.10)$$

and

$$\frac{\dot{R}}{R^n} = C \quad \text{(a constant)} \qquad (2.11)$$

Intergrating (2.11) we have

$$ct = \frac{R^{(1-n)}}{(1-n)} + A$$

As $t \to 0$, R also tends to zero and so we must have $n < 1$ and $A = 0$ then we get.

$$R = \{(1-n)\, Ct\}^{1/(1-n)} \quad , \; n < 1 \quad (2.12)$$

With the help of (2.10) (2.11) and (2.12) the equation (2.9) becomes.

$$-\left(\frac{p_o f_1^1 + H_o g^1}{p_o \psi_1} + \phi_1 \phi_1^1\right)(n\phi_1 - \eta\phi_1^1) - 1 = C$$

(2.13)

puting (2.5), (2.6), (2.7), (2.8), and (2.10) in the equation (2.2), (2.3) and (2.4), we get

$$\frac{\psi_1}{\eta}(\eta\phi_1^1 + 2\phi_1) = \psi_1(\eta c - \phi_1) \quad (2.14)$$

$$c(\eta n - \eta^2 \frac{g_1^1}{g_1}) + \eta\phi_1 \frac{g_1^1}{g_1} + n\phi_1^1 + 2\phi_1 = 0 \quad (2.15)$$

$$c(2nf_1 - \eta f_1^1) + \phi_1 f_1^1 = \gamma f_1 (\phi - \eta c) \frac{\psi_1^1}{\psi_1} \quad (2.16)$$

Also the assumption (2.5), (2.6), (2.7) and (2.8) become

$$P = p_o R^{2n} f_1(\eta)$$

$$\rho = p_o \psi_1(\eta) \quad (2.17)$$

$$U = R^n \phi_1(\eta)$$

$$H = H_o R^n g_1(\eta)$$

where the parameter n remains arbitrary and C is an absolute constant.

4. INTITIAL CONDITIONS

Let the explosion take place at a point at

$t = o$. Then at $t = o$

$U \to \infty$ for $x \to o$

$U = o$ for $x \neq o$

and, as obtained, p, ρ, H and U are all finite for $t > 0$ everywhere in $x \leq R$.

We consider the case for $n = -3/2$ (a choice that is helpful in simplification without causing any loss in generallity and that (2.12) given

$$R = \beta\, t^{2/5} \qquad (2.18)$$

which defines the shock radius at any time t β beingan absolute constant. The shock velocity is given by

$$U = \frac{dR}{dt} = \frac{2}{5}\left(\frac{R}{t}\right) \qquad (2.19)$$

Using (2.18) and changing the notations slightly for convenience we write the assumptions (2.17) as

$$\rho = f_1(\xi) \qquad , p = t^{-6/5} f_3(\xi) \qquad (2.20)$$

$$U = t^{-3/5} f_2(\xi) \quad , H = t^{-3/5} f_4(\xi)$$

where

$$\xi = x\, t^{-2/5} \qquad (2.21)$$

We shall now consider the energy equation . If

$$\frac{E}{\rho} = \left(\frac{1}{2} U^2 + \frac{p}{\rho(\gamma-1)} + \frac{H^2}{2\rho} \right) \qquad (2.22)$$

denote the total energy per unit mass and . If

$$\frac{I}{\rho} = \left(\tfrac{1}{2} U^2 + \frac{\gamma p}{\rho(\gamma-1)} + \frac{H^2}{\rho} \right) \qquad (2.23)$$

donote the enthalpy per unit mass then

$$\frac{\partial E}{\partial t} = \rho U \frac{\partial U}{\partial t} + \tfrac{1}{2} U^2 \frac{\partial \rho}{\partial t} + H \frac{\partial H}{\partial t} \quad \frac{\gamma p}{p(\gamma-1)}$$

$$\left(\frac{\partial p}{\partial t} + U \frac{\partial \rho}{\partial x} \right) - \frac{U}{(\gamma-1)} \frac{\partial p}{\partial x} \qquad (2.24)$$

where the value of $\frac{\partial p}{\partial t}$ has been substituted from (2.4).

Also,

$$\frac{1}{x^2} \frac{\partial (x^2 U I)}{\partial x} = \tfrac{1}{2} U^3 \frac{\partial \rho}{\partial x} + U^2 \rho \frac{\partial U}{\partial x} + \frac{\gamma U}{(\gamma-1)} \frac{\partial p}{\partial x}$$

$$+ 2HU \frac{\partial H}{\partial x} + \frac{\rho U^3}{x} + \frac{2\gamma p U}{(\gamma-1)x} + \frac{2UH^2}{x}$$

$$E = \tfrac{1}{2} f_1(\xi)\, t^{-6/5}\, f_2^2(\xi) + \frac{1}{(\gamma-1)}\, t^{-6/5}\, f_3(\xi) + \tfrac{1}{2}\, t^{-6/5}\, f_4^2(\xi)$$

or,

$$E = t^{-6/5}\, f(\xi) \qquad (2.28)$$

where

$$f(\xi) = \tfrac{1}{2} f_1(\xi) f_2^2(\xi) + \frac{1}{(\gamma-1)} f_3(\xi) + \tfrac{1}{2} f_4^2(\xi)$$

so that,

$$\frac{\partial E}{\partial x} = f'(\xi)\, t^{-8/5}$$

and,

$$\frac{\partial E}{\partial t} + \frac{6E}{5t} + \frac{2x}{5t}\frac{\partial E}{\partial x} = 0 \qquad (2.30)$$

With the help of (2.30) and (2.27) we get,

$$\frac{\partial(x^2 U_1)}{\partial x} = \frac{\partial}{\partial x}\left(\frac{2x^3 E}{5t}\right) \qquad (2.31)$$

Integrating the above and putting the constant of integration zero as well as applying (2.19) we get,

$$+ \frac{1}{2} \rho U^2 \frac{\partial u}{\partial x} + \frac{\gamma p}{(\gamma-1)} \frac{\partial u}{\partial x} + H^2 \frac{\partial u}{\partial x} \qquad (2.25)$$

Therefore,

$$\frac{\partial E}{\partial t} + \frac{1}{x^2} \frac{\partial (x^2 U I)}{\partial x} = \rho U \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{H}{\rho} \frac{\partial H}{\partial x} \right)$$

$$+ \frac{1}{2} u^2 \left(\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial x} + \rho \left(\frac{\partial u}{\partial x} + \frac{2u}{x} \right) \right)$$

$$+ \frac{\gamma p}{\rho(\gamma-1)} \left(\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial x} + \rho \left(\frac{\partial u}{\partial x} + \frac{2u}{x} \right) \right)$$

$$+ H \left(\frac{\partial H}{\partial t} + u \frac{\partial H}{\partial x} + H \left(\frac{\partial u}{\partial x} + \frac{2u}{x} \right) \right) \qquad (2.26)$$

By virtue of equation (2.1), (2.2) and (2.3) we obtain the energy relation as

$$\frac{\partial E}{\partial t} + \frac{1}{x^2} \frac{\partial (x^2 U I)}{\partial x} = 0 \qquad (2.27)$$

With the help of (2.20), we have from (2.22)

$$\frac{u}{U} = \frac{xE}{ER} \qquad (2.32)$$

The equation (2.32) can also be written as

$$U' = x' \left(\frac{\frac{1}{2} U^2 u^2 + \dfrac{p}{\rho(\gamma-1)} + \dfrac{H^2}{2\rho}}{\frac{1}{2} U^2 U^2 + \dfrac{\gamma p}{\rho(\gamma-1)} + \dfrac{H^2}{\rho}} \right)$$

where,

$$u = Uu' \ , \ x = Rx' \qquad (2.33)$$

Therefore,

$$\frac{p}{\rho} = \frac{1}{(\gamma U' - x')} \left\{ c_1 U'^2 (x' - U') + \frac{c_1 H^2}{\rho U^2} (x' - 2U') \right\}$$

or

$$\frac{p}{\rho} = \frac{1}{(\gamma U' - x')} \left\{ c_1 U'^2 (x' - U') + c_2 \rho (x' - 2U') \right\} \qquad (2.34)$$

where c_1 and c_2 are function of time and $H = b\rho$ as a consequence of (2.2) and (2.3). From (2.2) and (2.3) we obtain,

$$\frac{1}{p}\frac{\partial p}{\partial x} - \frac{(\gamma-1)}{\rho}\frac{\partial \rho}{\partial x} = -\frac{1}{Up}\frac{\partial p}{\partial t} + \frac{(\gamma-1)}{U\rho}\frac{\partial \rho}{\partial t} - \frac{1}{U}\frac{\partial u}{\partial x} - \frac{2}{x}$$

(2.35)

The specified dependence on t make the problem, in effect dependent on a single independent variable x, because,

$$\frac{\partial \rho}{\partial t} = -\frac{x}{R}\, U\, \frac{\partial \rho}{\partial x} \qquad (2.36)$$

$$\frac{\partial p}{\partial t} = -\frac{3pU}{R} - \frac{xU}{R}\frac{\partial p}{\partial x} \qquad (2.37)$$

$$\frac{\partial H}{\partial t} = -\frac{HU}{R} - \frac{xU}{R}\frac{\partial H}{\partial x} \qquad (2.38)$$

By putting (2.36) and (2.37) in the equcation (2.35) we have

$$\frac{1}{p}\frac{dp}{dx} - \frac{(\gamma-1)}{\rho}\frac{d\rho}{dx} = -\frac{2}{x} - \left(\frac{\frac{1}{R} - \frac{1}{U}\frac{dU}{dx}}{\frac{x}{R} - \frac{U}{V}} \right) \qquad (2.39)$$

On integrating and substituting (2.33) this become,

$$\frac{p}{\rho^{(\gamma-1)}} = \frac{C_3\,(x'-U')^{-1}}{x'^2} \qquad (2.40)$$

Eliminating p between (2.34) and (2.40) and dropping primes for the sake of convenience, we obtain the eqation

$$\rho^{(3-\gamma)}(x-2U) + D_1 U^2 (x-U)\,\rho^{(2-\gamma)}$$

$$= \frac{D_2 \, (\gamma U - x)}{x^2 \, (x-U)} \qquad (2.41)$$

to determine ρ ; D_1 and D_2 being constant on time . In the same eliminating ρ between (2.39) and (2.40) we have,

$$p^{\left(\frac{\gamma-2}{\gamma-1}\right)} - p^{\left(\frac{1}{\gamma-1}\right)} A_1 \frac{(x-2U)}{(\gamma U - x)} \{x^2 \, (x-U)\}^{2/(\gamma-1)}$$

$$= A_2 \frac{U^2 \, x^{1/(\gamma-1)} \, (x-U)^{\gamma/(\gamma-1)}}{(\gamma U - x)} \qquad (2.42)$$

which determines p ; A_1 , A_2 are constants depending on time

Again as a consequence of (2.2) (2.3) and (2.4) we have,

$$H^{(2-\gamma)} (x-2U) + B_1 U^2 (x-U) \, H^{(2-\gamma)}$$

$$= \frac{B_2 \, (\gamma U - x)}{x^2 \, (x-U)} \qquad (2.43)$$

where B_1 and B_2 are constants depending on time. Henceforth for simplicity we write total derivatives in place of partial ones From equation (2.2) on using (2.33) and then dropping the primes we get

$$\frac{1}{\rho} \frac{d\rho}{dx} = \frac{1}{(x-U)} \left(\frac{dU}{dx} + \frac{2U}{x} \right) \qquad (2.44)$$

Differentiating (2.41) with respect to x and eliminating $\frac{dp}{dx}$ with the help of (2.44) we have

$$\rho(3-\gamma)\left\{\frac{(3-\gamma)}{(x-U)}\left(\frac{dU}{dx}+\frac{2U}{x}\right)(x-2U)+\left(1-\frac{2dU}{dx}\right)\right\}$$

$$+ D_1\, p^{(2-\gamma)}\left\{(2-\gamma)U^2\left(\frac{dU}{dx}+\frac{2U}{x}\right)+\frac{dU}{dx}(2x-3U^2)+U^2\right\}$$

$$= D_2\,\frac{\left(\gamma\frac{dU}{dx}-1\right)x^2(x-U)-(\gamma U-x)(3x^2-2xU-x^2)\frac{dU}{dx}}{x^4\,(x-U)^2}$$

(2.45)

The equation (2.45) and (2.44) express relationship between U and x. Then (2.41),(2.42) and (2.43) express ρ,p and H in terms of x and therefore, given the required solution.

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CHAPTER - III

KLIMSHIN'S EFFECT IN POINT EXPLOSION IN STELLAR INTERIORS

1. INTRODUCTION

In chapter II, we have discussed the solutions of the equation of spherical blast wave produced by a strong point explosion . In this section we consider the same point explosion in stellar bodies . Since the temperature of the material of the stellar bodies is very high , the radiation effect cannot be ignored . The problem refrred to [1],[2],[3], in chapter II have been discussed without any radiation effect in non-gravitating system while the aim in this part is to discuss the equation for the propagation of a radiative blast wave produced by a sudden point explosion in self - gravitating system such as stars . We have taken the same similarity solution as in chapter II and the medium has been assumed to be a perfectly conducting plasma with radiative parameter independent of magnetic field . The disturbance is bounded on the outside . In this case also the viscosity and heat conduction are neglected and it is assumed that the flow is isentropic along a stream line . Lee [4] discussed the model of blast wave modle to account for intiation energy.

2. EQUATIONS GOVERNING THE FLOW AND FIELD IN SELF-GRAVITATING BODIES.

The equation of motion , continuity, energy and the field equation in the case of a radiative gas are,

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{H}{\rho}\frac{\partial H}{\partial r} + \frac{Gm}{r^{2}} = 0 \qquad (3.1)$$

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\left(\frac{\partial u}{\partial r} + \frac{2u}{r}\right) = 0 \qquad (3.2)$$

$$\frac{\partial (E+E_H)}{\partial t} + u\frac{\partial (E+E_H)}{\partial r} + (p+p_H)\left(\frac{\partial}{\partial t}\left(\frac{1}{\rho}\right) + u\frac{\partial}{\partial r}\left(\frac{1}{\rho}\right)\right)$$

$$+ \frac{1}{\rho r^{2}}\frac{\partial (F r^{2})}{\partial r} = 0 \qquad (3.3)$$

$$\frac{\partial H}{\partial t} + u\frac{\partial H}{\partial r} + H\left(\frac{\partial u}{\partial r} + \frac{2u}{r}\right) = 0 \qquad (3.4)$$

where

$$E = E_m + E_R \quad , \quad p = p_m + p_R \qquad (3.5)$$

$$E_H = \frac{H^{2}}{2\rho} \qquad p_H = \frac{H^{2}}{2}$$

the suffixes M,R and H, attached to a symbol denote expressions for material radiation and magnetic term respectively. The quantities u, p, and ρ are radial velocity pressure and density at a

distance r at any time from the point of explosion the magnetic field has components $(0,0,H)$, F is the radiation flux, G is the gravitational constant and m is the mass within the shock front at any time t such that

$$\frac{\partial m}{\partial r} = 4\pi\rho\, r^2 \qquad (3.6)$$

we have,

$$E_m = \frac{p_m}{\rho(\gamma-1)}, \quad E_R = \frac{3p_R}{\rho} \qquad (3.7)$$

and

$$F = -\frac{c}{\varkappa\rho}\,\frac{dp_R}{dr} \qquad (3.8)$$

where $\varkappa$ is the coefficient of opacity, c is the velocity of light

We assume as in [5]

$$p_m = zp, \quad p_R = (1-z)p, \quad (0 < Z < 1)$$

so that,

$$E = \frac{p}{\rho(k-1)} \qquad (3.9)$$

where K is called Klimishin's coefficiint and is given by [5]

$$K = \frac{4(\gamma-1) + Z(4-3\gamma)}{3(\gamma-1) + Z(4-3\gamma)} \qquad (3.10)$$

γ being the usual ratio of specific heats . With the help of (3.9),(3.2) and (3.4) , (3.3) can be written as ,

$$\frac{\partial p}{\partial t} + U \frac{\partial p}{\partial r} + Kp \left(\frac{\partial U}{\partial r} + \frac{2U}{r} \right) + \frac{(K-1)}{r^{2}} \frac{\partial (Fr^{2})}{\partial r} = 0 \qquad (3.11)$$

Let the motion be assumed to be confined within the shock front at $r = R(t)$. Then , the velocity of the shock moving outwards is given by.

$$V = \frac{dR}{dt} \qquad (3.12)$$

<u>3. SELF SIMILAR EXISTANCE</u>

The following similarity forms are used for flow and field variables.

$$p = p_0 R^m f_1(\eta) \qquad (3.13)$$

$$\rho = \rho_0 \psi_1 (\eta) \qquad (3.14)$$

$$U = R^n \phi_1 (\eta) \qquad (3.15)$$

$$H = H_0 R^k g_1 (\eta) \qquad (3.16)$$

$$F = F_0 R^l h_1 (\eta) \qquad (3.17)$$

$$m = m_0 R^b m_1 (\eta) \qquad (3.18)$$

where $\eta = r/R$ is a non dimensional radial variable f_1 , ψ_1, ϕ_1, g_1 , h_1 and M_1 are functions of η only and the suffix zero denotes quantities in the undeisturbed state . Putting the equations (3.1) to (3.4) and (3.6) into their similarity form,we

gor,

$$\frac{\dot{R}}{R^{n}} (n\phi_1 - \eta\phi_1') + \phi_1 \phi_1' = - \frac{1}{\rho_0\psi_1}$$

$$(P_0 R^{(m-4n)} F_1' + H_0 R^{(2k-2n)} g_1' + m_0 R^{(b-2n-1)} m_1 \rho_0 \phi_1 \quad (3.19)$$

$$\frac{\phi_1}{\eta} (\eta \phi_1' + 2\phi_1) = \phi_1' (\eta \frac{\dot{R}}{R^{n}} - \phi_1) \quad (3.20)$$

$$\frac{\dot{R}}{R^{n}} (m f_1 - \eta f_1') = k (\phi_1' f_1 + \frac{2\phi_1 f_1}{\eta})$$

$$+ \frac{(k-1) f_0}{\rho_0} R^{(l-m-n)} (m_1'\eta + \frac{2m_1}{\eta}) \quad (3.21)$$

$$\frac{\dot{R}}{R^{n}} (\eta k - \eta^{2} \frac{g_1'}{g_1}) + \eta \phi_1 \frac{g_1'}{g_1} + \eta \phi_1' + 2\phi_1 = 0 \quad (3.22)$$

$$m_1' = - \frac{4\pi\rho_0}{m_0} R^{-b+3} \psi_1 \eta^{2} \quad (3.23)$$

In order that all the unknowns may be expressible as fuctions of η along , the following relations must be fulfilled.

$$m = 2n , k = n , l = 3n , b = (2n+1) \quad (3.24)$$

and

$$\frac{\dot{R}}{R^n} = C \qquad (3.25)$$

Integrating (3.25) we get,

$$Ct = \frac{R^{(1-n)}}{(1-n)} + A$$

n being an arbitrary parameter and C an absolute constant. As t ---> o and R ---> o and so we must have n < 1 and A = o . Then shock radius R is given as

$$R = \{ (n-1)\, Ct \}^{\frac{1}{(1-n)}} \qquad n < 1 \qquad (3.26)$$

With the help of (3.24) ,(3.25) and (3.36) the equation (3.19) to (3.23) become.

$$C (n \phi_1 - \eta \phi_1') + \phi_1 \phi_1' = - \frac{1}{\rho_o \psi_1}$$

$$(p_o f_1' + H_o g_1' + m_o m_1 \rho_o \psi_1) \qquad (3.27)$$

$$\frac{\psi_1}{\eta} (\eta \phi_1' + 2\phi_1) = \psi_1' (\eta C - \phi_1) \qquad (3.28)$$

$$C (2n f_1 - \eta f_1') = k (f_1 \phi_1' + \frac{2\phi_1 f_1}{\eta})$$

$$+ \frac{(k-1)F_o}{P_o} \left(n_1' \eta + \frac{2n_1}{\eta} \right) \qquad (3.29)$$

$$c \left(n\eta - \frac{g_1' \eta^2}{g_1} \right) + \eta \frac{\phi_1 g_1'}{g_1} + n \phi_1' + 2\phi_1 = 0$$

(3.30)

and the similarity transformation (3.13) to (3.18) become

$$p = p_o R^{2n} f_1(\eta) \qquad \rho = \rho_o \psi_1(\eta)$$

$$U = R^n \phi_1(\eta) \qquad H = H_o R^n g_1(n) \qquad (3.31)$$

$$F = F_o R^{2n} n_1(\eta) \qquad m = m_o R^{(2n+1)} m_1(\eta)$$

4. INITIAL CONDITIONS AND SOLUTIONS

Let the explosion take place at a point at $t = o$. then , at $t = o$,

$U \rightarrow \infty$ for $r \rightarrow o$

$U = o$ for $r \neq o$

and also as obtained earlier $R \rightarrow o$. For any admissible solution , p, ρ , H and U are all finite for , $t \rightarrow o$ everywhere in $r \leq R$ As in the chapter II we consider the case for $n = -3/2$ By putting the value of n in (3.26) we get the shock radius R as

$$R = \beta \, t^{2/5} \qquad (3.32)$$

β is an absolute constant. From (3.32) the shock velocity is then given by

$$V = \frac{dR}{dt} = \frac{2}{5}\left(\frac{R}{t}\right) \qquad (3.33)$$

Again by using (3.32) the similarity transformations can be written as.

$$\rho = f_1(\xi) \qquad U = t^{-3/5} f_2(\xi)$$

$$p = t^{-6/5} f_3(\xi) \qquad H = {}^{-3/5} f_4(\xi)$$

$$F = t^{-9/5} f_5(\xi) \qquad m = t^{-4/5} f_6(\xi) \qquad (3.34)$$

$$\xi = r\, t^{-2/5} \qquad (3.35)$$

The energy equa tion can be written in the form,

$$\frac{\partial E}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r} r^2 (UI_T + F) = o \qquad (3.36)$$

$$E_T = \tfrac{1}{2}\rho U^2 + \frac{p}{(k-1)} + \frac{H^2}{2} - \frac{Gm\rho}{r} \qquad (3.37)$$

$$I_T = \tfrac{1}{2}\rho U^2 + \frac{kp}{(k-1)} + H^2 - \frac{Gm\rho}{r} \qquad (3.38)$$

putting (3.37) into their similarity form we get,

$$E_T = \tfrac{1}{2} f_1(\xi_j)\ t^{-6/5}\ f_2^2(\xi_j) + \frac{1}{(k-1)}\ t^{-6/5}$$

$$f_3(\xi_j) + \tfrac{1}{2}\ t^{-6/5}\ f_4(\xi_j) - \frac{t^{-6/5} f_6(\xi_j) f_1(\xi_j)}{\xi_j} \qquad (3.39)$$

or $$E_T = t^{-6/5}\ f(\xi_j)$$

where

$$f(\xi_j) = \tfrac{1}{2}\ f_1(\xi)\ f_2^2(\xi_j) + \frac{1}{(k-1)}\ f_3(\xi_j) + \tfrac{1}{2}\ f_4(\xi_j)$$

$$- \frac{f_6(\xi_j) f_1(\xi_j)}{\xi_j} \qquad (3.40)$$

From (3.39) we have,

$$\frac{\partial E_T}{\partial r} = f'(\xi_j)\ t^{-6/5} \qquad (3.41)$$

$$\frac{\partial E_T}{\partial t} = -\frac{6}{5}\ t^{-11/5}\ f(\xi_j) - t^{-6/5}\ f'(\xi_j)\ \frac{2}{5}\ r\ t^{-7/5} \qquad (3.42)$$

With the help of (3.39) , (3.41) , (3.42) we get ,

$$\frac{\partial E_T}{dt} + \frac{6}{5t} E_T + \frac{E_T}{5t} \frac{\partial E_T}{\partial r} = 0 \quad (3.43)$$

$$\frac{\partial E_T}{\partial t} + \frac{2}{5t r^2} \frac{\partial r^2 E_T}{\partial r} = 0 \quad (3.44)$$

From (4.5) and (3.44) we have,

$$\frac{\partial}{\partial r} (r^2 (UI_T + F)) = \frac{\partial}{\partial r} \left(\frac{2r^2 E_T}{5t} \right) \quad (3.45)$$

or

$$(UI_T + F) = \frac{V_T E_T}{R}$$

where we have taken the constant of integration to be zero and the value of V is substituted from (3.33).The equation (3.45) can also be written as

$$U' = \frac{\partial (\tfrac{1}{2} V^2 \rho U^2 + \frac{P}{(k-1)} + \frac{H^2}{2} - \frac{Gmp}{Rx})}{(\tfrac{1}{2} V^2 \rho U^2 + \frac{kp}{(k-1)} - \frac{Gmp}{Rx}} - \frac{F}{VI_T} \quad (3.46)$$

where

$$U = VU' \quad , \quad \tau = Rx$$

From (3.46) we have,

$$\frac{p}{\rho} = \frac{(k-1)}{(kU'-x)} \{ C_1 U'^2 (x-U') + C_2\rho (x-2U')$$

$$+ \frac{C_3 m}{x} (U'-x) - C_4 F \} \qquad (3.48)$$

where C_1, C_2, C_3 and C_4 are function of time and $H = C\rho$ as a consequence of (3.2) and (3.3) . From (3.2) and (3.11) we obtain

$$\frac{1}{p}\frac{\partial p}{\partial \tau} - \frac{(k-1)}{\rho}\frac{\partial \rho}{\partial r} = -\frac{1}{pU}\frac{\partial p}{\partial t} + \frac{(k-1)}{\rho U}\frac{\partial \rho}{\partial t} - \frac{(k-1)}{r^2}\frac{\partial (F r^2)}{\partial r}$$

$$- \frac{1}{U}\frac{\partial U}{\partial \tau} - \frac{2}{\tau} \qquad (3.49)$$

by putting the value of Σ and V in (3.34) we get the ralations,

$$\frac{\partial \rho}{\partial t} = \frac{\tau V}{R}\frac{\partial \rho}{\partial \tau} \qquad (3.50)$$

$$\frac{\partial p}{\partial t} = \frac{-3pV}{R} \frac{\tau V}{R}\frac{\partial p}{\partial \tau} \qquad (3.51)$$

By using (3.50) and (3.51) we can write equation (3.49)

$$\frac{1}{p}\frac{\partial p}{\partial r} - \frac{(k-1)}{\rho}\frac{\partial \rho}{\partial r} = -\frac{2}{r}\,\frac{\left(\frac{1}{V}\frac{\partial U}{\partial r} - \frac{1}{R}\right)}{\left(\frac{U}{V} - \frac{r}{R}\right)}$$

$$- \frac{(k-1)\,U}{V\left(\frac{U}{V} - \frac{r}{R}\right)r^2}\,\frac{\partial (F r^2)}{\partial r} \qquad (3.52)$$

Integrating this we get,

$$\frac{p}{\rho^{(k-1)}} = \frac{C_5}{r^2\left(\frac{U}{V} - \frac{r}{R}\right)}\exp\int \frac{(k-1)\,U}{V\left(\frac{U}{V} - \frac{r}{R}\right)r^2}\,\frac{\partial (F x^2)}{\partial r}\,dr \qquad (3.53)$$

Substituting (3.47) in (3.53) we have,

$$\frac{p}{\rho^{(k-1)}} = \frac{C_5}{x^2\,(U' - x)}\exp\int \frac{(k-1)\,U'}{(U' - x)\,x^2}\,\frac{\partial (F x^2)}{\partial x}\,dx \qquad (3.54)$$

Eliminating P between (3.54) and (3.48) and deropping the primes, we obtain the equation,

$$p^{(3-k)}\ (\tau-2U) + p^{(2-k)}\ (D_1\ U^2\ (\tau-U) + \frac{D_2 m}{\tau}(U-\tau) - D_3 F$$

$$= \frac{D_4\ (kU-\tau)}{\tau^2(U-\tau)} - D_5(kU-\tau)\ \exp\int \frac{(k-1)U}{(U-\tau)\tau^2}\ \frac{d(F\tau^2)}{d\tau}\ dx$$

which determines p. For simplicity we can write total derivatives in place of partial ones. D_1, D_2, D_3, D_4 and D_5 being constants depending upon time . Similarly eliminating p between (3.54) and (3.48) we can get,

$$p^{\left(\frac{k-2}{k-1}\right)} = \frac{A_1\ (\tau - 2U)\ p^{(1/k-1)}}{(kU-\tau)\cdot\left\{\frac{A_2}{\tau^2(U-\tau)} - \exp\int \frac{(k-1)U}{(U-\tau)\tau^2}\ \frac{d(F\tau^2)}{d\tau}\ dx\right\}^{(2/k-1)}}$$

$$= \frac{(A_3\ U^2(\tau-U) - A4\ m/\tau\ (\tau-U) - A_5 F)}{(kU-\tau)}$$

$$\times\left\{\frac{A_6}{\tau^2(U-\tau)}\quad \exp\int \frac{(k-1)U}{(U-\tau)\tau^2}\ \frac{d(F\tau^2)}{d\tau}\ d\tau\right\}^{-(1/k-1)} \qquad (3.56)$$

which determines p. A_1, A_2, A_3, A_4, A_5 and A_6 are constants depending on time

As a consequence of (3.2) (3.4) and (3.55) we write

$$\rho^{(3-k)}(r-2U) + \rho^{(2-k)}\left\{B_1U^2(r-U) + \frac{B_2 m}{r}(U-r) - B_3F\right\}$$

$$= B_4\,\frac{(kU-r)}{r^2(U-r)} - B_5\,(ku-r)\exp\int\frac{(k-1)U}{(U-r)r^2}\,\frac{d(Fr^2)}{dr}\,dr \qquad (3.57)$$

where $B_1, B_2, B_3, B_4,$ and B_5 are constants depending on time.

From equation (3.2) on using (3.47) and (3.48) we get ,

$$\frac{1}{\rho}\left(\frac{d\rho}{dr}\right) = \frac{1}{(r-U)}\left(\frac{dU}{dr} + \frac{2U}{r}\right) \qquad (3.58)$$

Differentiating (3.55) with respect to r and then using (3.58) we get ,

$$\rho^{(3-k)}\left\{\frac{(3-k)\,(r-2U)\left(\frac{dU}{dr}+\frac{2U}{r}\right)}{(r-U)} + \left(1-\frac{2dU}{dr}\right)\right\}$$

$$+\,\rho^{(2-k)}\,\frac{(2-k)\left(\frac{dU}{dr}+\frac{2U}{r}\right)}{(r-U)}\,\{B_1U^2(r-U))$$

$$+ \frac{D_2 m(U-\tau)}{\tau} - D_3 F \Big\} + \frac{d}{d\tau} \Big\{ D_1 U^2 (\tau - U) + \frac{D_2 m(U-\tau)}{\tau} - D_3 F \Big\}$$

$$\frac{d}{d\tau} \Big[\frac{D_4 (kU-\tau)}{\tau^2 (U-\tau)} - D_5 (kU-\tau) \Big]_{\exp} \int \frac{(k-1)\,U}{(U-\tau)\,\tau^2} \frac{d(F\tau^2)}{d\tau}\, d\tau \Big]$$

(3.59)

The equation (3.59) and (3.55) express relationship between U and r then (3.55) , (3.56) and (3.57) express ρ,p and H in terms of r and therefore, given the required solution.

In the absence of any radiation effects,the Klimshin's coefficient K becomes the usual adiabatic exponent τ and then , the solution refferred above agrees with the corresponding solution in chapter II

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CHAPTER IV

A DISCUSSION OF SINGULARITIES ON EXPLODING RADIATIVE SPHERICAL DETONATIONS

INTRODUCTION.

Roger A strehlow (1) described how simple acoustic sourse theory may be opplied to determine maximun pressthe by deflagration of non spherical detonations.Kynch [2] Taylor [3] studied the propagation exploding shock waves in a gravitational free system by assuming the undisturbed density to vary according to some inverse power of distance from the centre of explosion.They however neglected counter pressure and used similarity concepts to simplify the analysis . An special case of the problem was taken by Taylors where counter pressure is assumed to be signicant [4]. But,if counter pressure is also taken into account the problem no longer remains self similar demanding there by the use of numerical methods for its solution . Sedov [4] therefore took into account counter pressure but assumed uniform density in the undisturbed state to avoid the use of the numericial methods. Verma [5]studied in conducting gases the propagation of a cylindrical shock produced on account of instantaneous energy release along a straight line by assuming the density of the undisturbed state to vary as r^{α} , r being the distance from the axis of explosion . The aim in this part of the Chapter is to study the propagation of a exploding detonations produced on account of an instantaneous energy release from the point of explosion in ordinary gases where radiation effect have been taken into account . As in [6] the density in the undisturbed state is taken to vary as r^{α},r being the distance from the point

of explosion. As established subsequently the mass and pressure be positive inthe equilibrium state, the choice of α is restricted between 0 and 3. The location of the point where singularity occurs in the course of integration has been discussed.

2. FORMULATION OF THE PROBLEM

The equations governing the flow behind a spherical shock wave are [7],

$$\rho\left(\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial r}\right) + \frac{\partial p}{\partial r} = 0 \qquad (4.1)$$

$$\frac{\partial \rho}{\partial t} + U\frac{\partial \rho}{\partial r} + \rho\left(\frac{\partial U}{\partial r} + \frac{2U}{r}\right) = 0 \qquad (4.2)$$

$$\frac{\partial E}{\partial t} + U\frac{\partial E}{\partial r} + p\left(\frac{\partial}{\partial t}\right)\left(\frac{1}{\rho}\right) + pU\frac{\partial}{\partial r}\left(\frac{1}{\rho}\right) + \frac{1}{\rho r^2}\frac{\partial (F r^2)}{\partial r} = 0 \quad (4.3)$$

where

$$E = E_m + E_R \text{ and } p = p_m + p_R \qquad (4.4)$$

E_T, E_m and E_R being the total energy, material energy and radiation energy respectively and P_T, P_m, P_R being the total pressur material pressure and radiation pressure. U and ρ are the velocity and density of the flow at distance r from the point of explosion and F is the radiation flux. We have,

$$E_M = \frac{p_m}{\rho(\gamma-1)} \quad , \quad E_R = \frac{3p_R}{\rho} \qquad (4.5)$$

Let us assume the variables as

$$p_m = Zp \quad , \quad p_R = (1-Z)p \quad , \quad (o < Z < 1)$$

then

$$E = \frac{p}{\rho(k-1)} \qquad (4.6)$$

where

$$k = \frac{4\,(\gamma-1) + Z\,(4-3\gamma)}{3\,(\gamma-1) + Z\,(4-3\gamma)} \qquad (4.7)$$

With the help of (4.6) , (4.3) becomes

$$\frac{\partial p}{\partial t} + U\frac{\partial p}{\partial r} + kp\left(\frac{\partial U}{\partial r} + \frac{2U}{r}\right) + \frac{(k-1)}{r}\frac{\partial (Fr^2)}{\partial r} = o \qquad (4.8)$$

The energy equation (as given in Chapter (II)equation (3.36) is therefore,

$$\frac{\partial E}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}\, r^2\,(UI+F) = o \qquad (4.9)$$

where

$$E = \tfrac{1}{2}\rho U^2 + \frac{p}{(k-1)} \qquad (4.10)$$

and

$$I = \tfrac{1}{2}\rho U^2 + \frac{kp}{(k-1)} \qquad (4.11)$$

Let the motion be assumed to be confined within the shock front $r = R(t)$. The velocity of the shock wave moving outwards is the given by

$$V = \frac{dR}{dt} \tag{4.12}$$

Let p_1, ρ_1 be the values of p and ρ infront of the shock when the flow velocity is assumed to be zero. Also let these quantities just behind the shock be denoted by p_2, ρ_2 and U_2. Then the generalized Rankine-Hugoniot relations for this case, can be written as,

$$\rho_2 = \rho_1 \phi \tag{4.13}$$

$$U_2 = \frac{V(\phi - 1)}{\phi} \tag{4.14}$$

$$p_2 = p_1 [1 + \frac{2\nu_1 (k_1 - 1)}{(1+\phi - 2\nu_1 \mu)} \{ \frac{(\phi - \mu)}{(k - 1)} + \frac{F_s \phi}{m s a_1^2} \}] \tag{4.15}$$

where

$$\nu_1 = \frac{k_1}{(k-1)}, \quad \nu_2 = \frac{k_2}{(k_2 - 1)}$$

F_1 has been neglected in comparision to F_2 which is now written as F_s. We have also assumed $\frac{\nu_2}{\nu_1} = \mu$

(a constant), $\rho_1 V = -ms$, and,

$$V^{*} = \frac{2\phi}{(V-\phi)\,(1+\phi-2V\mu)(k-1)}\left\{\frac{a_1^2}{\ }(\phi-\mu) + \frac{F_s\phi}{m_s}\right\} \quad (4.16)$$

where

$$a_1 = \left(\frac{k_1\, p_1}{\rho_1}\right)^{\frac{1}{2}} \qquad (4.17)$$

is defined as a pseudo-sound velocity . From (4.14) it is evident that the velocity of the shock wave is greater than the velocity of the mass particles behind the shock . Hence, mass enveloped by the shock front at any time must be equal to the mass contained within the sphere of radius R in the undisturbed state . Let the density in the undisturbed state be given by

$$\rho_1(r) = \beta r^{-\alpha} \qquad (4.18)$$

where β and α are positive constants . The mass within the shock front is givenby

$$m = \int_0^R 4\pi.\rho\, r^2\, dr \qquad (4.19)$$

$$= \frac{4\pi\ \beta\ R^{(3-\alpha)}}{(3-\alpha)}$$

which is positive only when $o < \alpha < 3$.

3. SELF SIMILAR SOLUTIONS

In order to reduce the equations of flow to orinary differential equations we effect the following similarity

transformations,

$$u = r t^{-1} U(\eta) \qquad (4.20)$$

$$\rho = r^{b}\, \Omega(\eta)$$

$$p = r^{b+2}\, t^{-2}\, P(\eta)$$

$$F = r^{b+3}\, t^{-3}\, \bar{F}(\eta)$$

where

$$\eta = r^{-\lambda}\, t \qquad (4.21)$$

The parameter b and λ are so far free but shall be fixed later to suit certain physical requirements . Let the shock surface be determined by,

$$\eta = \eta_1 \qquad (4.23)$$

where η_1 is constant so that

$$\eta_1 = R^{-\lambda}\, t \qquad (4.24)$$

and

$$V = \frac{dR}{dt} = \frac{R}{\lambda t} \qquad (4.25)$$

The total energy within the shock front is then given by

$$E = \int_0^R 4\pi\rho\, r^2 \left(\tfrac{1}{2} u^2 + \frac{p}{\rho(k-1)} \right) dr$$

$$= \frac{4\pi}{\lambda} \int_{\eta_1}^{\infty} \left(\tfrac{1}{2}\, \Omega\, U^2 + \frac{P}{(k-1)} \right) \eta^{\frac{-(b+\lambda+5)}{\lambda}}\, t^{\,b+5-2\lambda} \qquad (4.26)$$

Let us assume that the explosion is instantaneous so that the total energy depends on t only through η. Then from (4.26) we have

$$(b+5) = 2\lambda \qquad (4.27)$$

defining

$$m_1^2 = \frac{V^*}{\frac{k_1 p_1}{\rho}} = \frac{V^*}{a_1^2} \qquad (4.28)$$

we have

$$\phi = \frac{[\mu(1+\nu_1 m_1^2)+\{g^*(m_1^2\nu_1+1)^2-(\frac{g}{m_1 g(k-1)}+\frac{2Fs}{a_1^2\, ms})\, m_1^2(2\nu_1\mu-1)\}^{\frac{1}{2}}]}{(m_1^2+\frac{2}{(k_1-1)}+\frac{2Fs}{a_1^2\, ms})} \qquad (4.29)$$

taking plus sign before the radical since ϕ must be essentially positive. The equation (4.13) to (4.15) can therefore be expressed in terms of M_1 and F_*. As a consequence of (4.25) and the similarity transformations (4.20) we can write,

$$U(\eta_1) = \frac{1}{\lambda}\frac{(\phi-1)}{\phi} \qquad (4.30)$$

$$\Omega(\eta_1) = \rho_1(R)^{-*} R\,\phi \qquad (4.31)$$

$$p(\eta_1) = \frac{P_1(R) - R^{-b}}{m_1^2 k_1 \lambda^2}$$

$$[1 - \frac{2\nu_1 (k_1 - 1)}{(1+\phi-2\mu\nu_1)} \{ \frac{(\phi-\mu)}{(k_1-1)} + \frac{Fs\phi}{msa_1^2}\} \qquad (4.32)$$

$$\bar{F}(\eta_1) = \frac{p_1 (R)^{-b}}{\lambda^3 m_1^2 \psi} [\frac{(\phi-\mu)}{(k-1)} \quad \frac{m_1^2 (1-\psi)(1+\phi-2\mu\nu_1)}{2\phi}] \qquad (4.33)$$

If ϕ is constant, M_1 will be constant by virtue of (4.16) and (4.29). Now since, η_1 is constant the left hand sides and hence the right hand sides of the relations (4.30) to (4.33) are also constant. This is ensured if,

$$\rho_1 (R)^{-b} = \beta_1 \quad \text{(a constant)} \qquad (4.34)$$

Comparing (4.34) with (4.18) we have,

$$b = -\alpha \quad , \beta_1 = \beta \qquad (4.35)$$

From (4.35) and (4.27) we get,

$$(5 - \alpha) = 2\lambda \qquad (4.36)$$

Thus the relations (4.35) and (4.36) fix the parameters b and λ for each value of α that is, corresponding to different density distributions in the undisturbed gas.

4. ANOTHER FORM OF SIMILARITY SOLUTION.

We now introduce a new independent variable

$$x = \frac{r}{R} \qquad (4.37)$$

This variable is related to η and η_1 by the relation

$$x = \left(\frac{\eta_1}{\eta} \right)^{2/(5-\alpha)} \tag{4.38}$$

Hence we assume the following expressions for the velocity, density , pressure and radiation flux [5],[6]

$$\begin{aligned} u &= V f(x) \\ \rho &= \rho_1(R)\, l(x) \\ p &= \frac{\rho_1(R)}{k} V^2 g(x) \\ F &= \frac{\rho_1(R)}{k} V^3 n(x) \end{aligned} \tag{4.39}$$

Introducing these in (4.1),(4.2), and (4.8) we have,

$$(x-f)\, f' = \frac{g'}{k l} + \frac{(\alpha-3)}{2} f \tag{4.40}$$

$$(x-f) \frac{l'}{l} = f' + \frac{2f}{x} - \alpha \tag{4.41}$$

$$(x-f) \frac{g'}{g} = k \left(f' + \frac{2f}{x} \right) - 3 + \frac{(k+1)}{g} \left(n' + \frac{2n}{x} \right) \tag{4.42}$$

then the total energy can be written in the form ,

$$E = \rho_1(R)V^2 \{ \tfrac{1}{2} f^2 + \frac{g}{k(k-1)} \} \tag{4.43}$$

$$= t^{-6/(5-\alpha)} \psi(x)$$

and hence the equation of energy (4.9) becomes,

$$-\frac{6}{(5-\alpha)} \frac{r^2 E}{t} - \frac{2r^3}{(5-\alpha)t} \frac{\partial E}{\partial r} + \frac{\partial}{\partial r}(r^2(UI+F))=0 \tag{4.44}$$

The inetrgration of obove equation gives

$$x^2 \{ fI + \frac{\rho_1(R)V^2 n}{k} \} - x^3 E = \frac{\theta(t)}{R^2 V} = \theta_1(t) \tag{4.45}$$

This is one of the intermediate integrals. Multiplyting (4.41) by

$\frac{3(k-1)}{(3-\alpha)}$ and then subtracting (4.42) from it we have

$$\frac{3(k-1)}{(3-\alpha)} \frac{I'}{I} - \frac{g'}{g} = \frac{(3-\alpha k)}{(3-\alpha)} \{ \frac{(1-f')}{(x-f)} + \frac{2}{x} \}$$

$$- \frac{(k-1)n}{g(x-f)} \{ \frac{n'}{n} + \frac{2}{x} \} \tag{4.46}$$

which on integration gives ,

$$f^{(3k-1)/(3-\alpha)} = Ag\{x^2(x-f)\}^{(3-\alpha k)/(3-\alpha)} (nx^2)^{(k-1)n/g(x-f)} + \exp\int \log nx^2 \frac{d}{dx}\left\{\frac{(k-1)n}{g(x-f)}\right\}dx \qquad (4.47)$$

where A is constant . This is the second integral . Now we put (4.39) into (4.13) to (4.16) to get the boundary conditions at $r = R$ i.e. $x = 1$.

$$f(1) = \frac{(\phi-1)}{\phi} \qquad (4.48)$$

$$l(1) = \phi \qquad (4.49)$$

$$g(1) = \frac{N}{m_1^2} \qquad (4.50)$$

$$n(1) = \frac{1}{\phi}\left[\frac{(\phi-\mu)}{(k-1)m_1^2} - \frac{(1-\phi)(1+\phi-2\mu\nu_1)}{2\phi}\right] \qquad (4.51)$$

where

$$N = \frac{1}{k_1}\left[1 - \frac{2\nu_1(k-1)}{(1+\phi-2\mu\nu_1)}\left\{\frac{(\phi-\mu)}{(k-1)} + \frac{Fs}{msa_1^2}\right\}\right] \qquad (4.52)$$

Introducing the auxiliary functions

$$D = \frac{1}{g}(x-f) \tag{4.53}$$

and

$$L = (k-1)(x-f)\frac{n}{g} \tag{4.54}$$

where

$$D(1) = \frac{m_1^2}{N} \tag{4.55}$$

and

$$L(1) = \frac{(k-1)m_1^2}{N\phi^{k}}\left\{\frac{(\phi-1)}{m_1^2(k-1)} - \frac{(1-\phi)(1+\phi-2)W_1}{2\phi}\right\} \tag{4.56}$$

the intermediate integral and the differential equations of the problem take the following form

$$L+BL \quad \frac{3(k-1)}{(3-\alpha)} \quad \frac{(6-\alpha k-\alpha)}{3(k-1)} \quad (x-f) \quad \frac{(3-\alpha)}{3(k-1)}\Big|_n \left\{\frac{(x-f)^2_m + L}{(x-f)^2}\right\}$$

$$(x^2)\left\{\frac{(3-\alpha k)}{3(k-1)} + \frac{L(3-\alpha)}{3(k-1)(x-f)^2}\right\} +$$

$$+\left(\exp\int \log nx^2 \quad \frac{d}{dx}\frac{L}{(x-f)^2}\,dx\right)^{(3-\alpha)/3(k-1)} \tag{4.57}$$

where

$$B = [\,(k-1)\,A\,]^{(3-\alpha)/3(k-1)} \qquad (4.58)$$

$$\frac{df}{dx}\{D(x-f)-1\} = \frac{2f}{x} + \frac{Df(\alpha-3)}{2} - 3 + \frac{L}{k(x-f)}\left(\frac{n'}{n} + \frac{2}{x}\right) \qquad (4.59)$$

$$\frac{dD}{dx} = \frac{D}{(x-f)}\{4-\alpha-kf' + \frac{2f}{x}(1-k) - \frac{L}{(x-f)}\left(\frac{n'}{n} + \frac{2}{x}\right) \qquad (4.60)$$

$$\frac{dL}{dx} = \frac{L}{(x-f)}\{4-(k+1)f' - \frac{2kf}{x} + \frac{n'}{n}(x-f) - \frac{L\left(\frac{n'}{n} + \frac{2}{x}\right)}{(x-f)} \qquad (4.61)$$

The equation (4.59) , (4.60) , and (4.61) when integrated numerically given f , D and L respectively . Then I and q can be obtained with the help of (4.47) and (4.53)

5.DISCUSSIONS

From (4.59) , (4.60) and (4.61) f , D and L become infinite at a point distant x from the point of explosion , where.

$$D\,(x-f) = 1 \qquad (4.62)$$

This reduces to

$$U = V\left(\frac{\tau}{R} - \frac{\sqrt{(k-1)}}{m} \right) \qquad (4.63)$$

That is the singularity occurs at a point where

$$\tau = R\left(\frac{U}{V} + \frac{\sqrt{(k-1)}}{m} \right) \qquad (4.64)$$

where $\frac{k}{(k-1)} = \gamma$ and $a = \left(\frac{\gamma p}{\rho}\right)^{\frac{1}{2}}$

On putting the value of k in (4.63) it gives

$$\tau = R\left(\frac{U}{V} + \frac{1}{m}\sqrt{\frac{(\tau-1)}{3(\gamma-1)+z(4-3\gamma)}} \right) \qquad (4.65)$$

In the case when $\tau = 7/5$ for a gas, (4.65) becomes

$$\gamma = R\left(\frac{U}{V} + \frac{1}{m}\sqrt{\frac{2}{(6-z)}} \right) \qquad (4.66)$$

and for a gas for which $\tau = 4/3$, (4.65) given

$$\gamma = R\left(\frac{U}{V} + \frac{1}{M\sqrt{3}} \right) \qquad (4.67)$$

We make, therefore, the following observation -

(a) The equation (4.66) and (4.67) indicate that for a gas where

$\gamma = 7/5$, the singularity occurs at a greater distance than for which $\gamma = 4/3$.

(B) It is obvious from the value of k given by (4.7) that while in the absence of radiation effects k is equal to γ, it is of interest to note that for a gas for which $\gamma = 4/3$, k also equal 4/3.

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CHAPTER V

COMPATIBILITY CONDITIONS FOR WEAK DISCONTINUITY IN THERMAL MEDIUM

1.INTRODUCTION :-

It has been observed that pressure density and velocity were continuous across a moving surface , while at least one of the first derivatves of these quantities with respect to the space coordinates were discontinuous , Thomas [1] called it a sonic wave of order one or simply a sonic wave and discussed its growth and decay . In the present chapter we have studied how these sonic discontinuities behave when a perfect gas is subjected to radiation . In the cours of discussion we have introduced a generalised form of Klimshin's cofficient obtained second and third order compatibility conditions and studied the growth and decay of plane and spherical waves [2].

2.EQUATIONS GOVERNING FLOW AND COMPATIBILITY CONDITIONS

Due to rerified medium we may omit viscosity and thermal conductivity , the differential equation governing the motion of a perfect gas , when radiation effects are taken into account are,

$$\rho \frac{\partial U_i}{\partial t} + \rho U_i U_{i,j} + p_{,i} = 0 \qquad (5.1)$$

$$\frac{\partial \rho}{\partial t} + U_i \rho_{,i} + \rho U_{i,i} = 0 \qquad (5.2)$$

$$\frac{\partial p}{\partial t} - \rho U_i \frac{\partial U_i}{\partial t} - \rho U_i U_j U_{i,j} + kp\, U_{i,i} + (k-1)\, F_{i,i} = 0 \qquad (5.3)$$

where

$$F_i = - \frac{Ac}{\tau \rho} (p_R)_{,i} \qquad (5.4)$$

and

$$p = p_m + p_R \qquad (5.5)$$

p_m and p_R being the material pressure and radiation pressure as in [2] . Equation (5.1) , (5.2) and (5.3) are referred to a system of rectangular Co-ordinates x_i a comma (,) as usual indicates partial derivative with respect to these co-ordinates . F_i is the radiation flux and k is the gernerallized Klimshin coefficient to be defined later. The material energy and radiation energy are given by $p_m/\rho(\gamma-1)$ and $3p_R/\rho$ respectively where γ is the ration of spacific heats . Assuming,

$$p_m = zp \quad , p_R = (1-z)p \quad , (0 < z < 1)$$

the total energy of the gas is given by

$$\frac{p}{\rho(k-1)} \qquad (5.6)$$

where

$$k = \frac{4(\gamma-1) + z(4-3\gamma)}{3(\gamma-1) + z(4-3\gamma)} \qquad (5.7)$$

it is easy to see that when $Z = 1$, that is, when radiation effects are not considered, the generalized co-efficient k' becomes equal to the usual adiabatic exponent γ [4]. we can write,

$$F_i = \frac{A}{\rho} p_{,i} \qquad (5.8)$$

where

$$A = -\frac{Ac(z-1)}{\tau} \qquad (5.9)$$

c being the veclocity of light, τ is the corfficirnt of opacity and A is a stefan Boltamann constsnt.

Let the moving surface be denoted by $\Sigma(t)$. Then if the discontinuity or Jump across the moving surface be indicated by a bracket [ ], we have, [5]

$$[p] = [\rho] = [U_i] = [F_i] = 0 \qquad (5.10)$$

over $\Sigma(t)$. We assume the regularity of the surface $\Sigma(t)$ and the existence of the limiting values of the functions and their derivatives as one approches this surface from each side. If G be the velocity of the moving surface, the following relations, called compatibility conditions of the first order, are satisfied.

$$[U_{i,j}] = \lambda_i V_j, \ [\frac{\partial u_i}{\partial t}] = -G\lambda_i \qquad (5.11)$$

$$[p_{,i}] = \xi_j V_i, \ [\frac{\partial p}{\partial t}] = -G\xi_j \qquad (5.12)$$

$$[p,_i] = \xi \nu_i \quad , \quad [\frac{\partial p}{\partial t}] = - G \xi \tag{5.13}$$

$$[F_i,_j] = \eta_i \nu_j \quad , \quad [\frac{\partial F_i}{\partial t}] = - G \eta_i \tag{5.14}$$

where the quantities ξ , ζ , λ_i and η_i are suitable functions defined over the surface $\Sigma(t)$, ξ and ζ being scalars. The quantities λ_i and η_i can be replaced by scalars λ and η since $\lambda_i = \lambda \nu_i$ and $\eta_i = \eta \nu_i$ where ν_i are the components of the unit normal ν to the surface $\Sigma(t)$

3.DYNAMICS OF MOVING DISCONTINUITY

From the equations (5.1) to (5.3) and the compatibility conditions (5.11) to (5.14) , we get,

$$\rho (U_n - G) \lambda_i + \xi \nu_i = 0 \tag{5.15}$$

$$(U_n - G) \zeta + \rho \lambda_i \nu_i = 0 \tag{5.16}$$

$$\rho (G - U_n) \lambda_i U_i - G\xi + kp \lambda_i \nu_i + (k-1)\eta = 0 \tag{5.17}$$

where U_n is the normal velocity. Multiplying (5.15) in turn by U_i and ν_i we get,

$$\rho (U_n - G) \lambda_i U_i + \xi U_n = 0 \tag{5.18}$$

$$\rho (U_n - G) \lambda_i \nu_i + \xi = 0 \tag{5.19}$$

Adding (5.17) and (5.18) we have,

$$\xi\,(U_n - G) + kp\,\lambda_i\,\nu_i + (k-1)\,\eta = 0 \qquad (5.20)$$

Multiplying (5.19) by kp (5.20) by $\rho(U_n - G)$ and subtracting we get ,

$$\xi\rho(U_n - G)^2 - \xi\,kp + (k-1)\,\eta\rho\,(U_n - G) = 0 \qquad (5.21)$$

As a consequance of (5.19) the equation (5.21) can be written as

$$\xi\,(\rho(U_n - G)^2 - kp - \frac{(k-1)\eta}{\lambda}) = 0 \qquad (5.22)$$

Let the sepeed S of the weak discontinuity defined by $S=(G-U_n)$ be different from zero. Then , if $\xi = 0$ on $\Sigma(t)$, it follows from (5.15) and (5.16) that $\psi = 0$ meaning thereby that the surface is out assumptions . Hence , $\xi \neq 0$ and we have from (5.22).

$$(U_n-G)^2 = \{ \frac{kp}{\rho} + \frac{(k-1)\eta}{\rho\lambda} \} \qquad (5.23)$$

Again from equation (5.16) , (5.19) and (5.23) we get ,

$$\psi = \frac{\rho\lambda}{(G-U_n)} \qquad (5.24)$$

$$\lambda = \frac{\xi}{p\,(G-U_n)} \qquad (5.25)$$

and

$$\eta = \left\{ (U_n - G)^2 - \frac{kp}{\rho} \right\} \frac{\rho\lambda}{(k-1)} \tag{5.26}$$

If we assume that the weak discontinuity is propagated into a gas at rest within which the total pressure p and density ρ are constant ; U_i = o on the surface $\Sigma(t)$ and hence the speed of propagation of the wave is given by

$$G^2 = \left(\frac{kp}{\rho} + \frac{(k-1)\eta}{\rho\lambda} \right) \tag{5.27}$$

and ,then equation (5.24) to (5.26) become

$$\varphi = \frac{\rho\lambda}{G} \tag{5.28}$$

$$\xi = \rho G \lambda \tag{5.29}$$

$$\eta = 2\rho B \lambda/(k-1) \tag{5.30}$$

where

$$2B = \left(G^2 - \frac{kp}{\rho} \right) \tag{5.31}$$

4. HIGHER ORDER COMPATIBILITY CONDITIONS

The conditions of compatibility of second and third orders of the quantities p , ρ, U_i and F_i are applicable in equations (5.54) to (5.58) . When G = constant , the compatibility conditions of the second order for velocity components U_i are given by,

$$[U_{i,jk}] = \bar{\lambda}_i \nu_j \nu_k + g^{\alpha\beta} \lambda_{i,\alpha} (\nu_j x_{k,\beta} + \nu_k x_{j,\beta})$$

$$- \lambda_i g^{\alpha\beta} g^{\sigma\tau} b_{\alpha\sigma} x_{j,\beta} x_{k,\tau} \qquad (5.32)$$

and

$$\left[\frac{\partial^2 U_i}{\partial x_j \partial t}\right] = (-G \bar{\lambda}_i + \frac{\delta \lambda_i}{\delta t})\nu_j - G f^{\alpha\beta} \lambda_{i,\alpha} x_{j,\beta} \qquad (5.33)$$

We corresponding conditions of compatibility for the functions p ρ and F_i are given by

$$[p_{,ij}] = \bar{\xi} \nu_i \nu_j + g^{\alpha\beta} \xi_{,\alpha}(\nu_i x_{j,\beta} + \nu_j x_{i,\beta})$$

$$- \xi g^{\alpha\beta} g^{\sigma\tau} b_{\alpha\sigma} x_{i,\beta} x_{j,\tau} \qquad (5.34)$$

$$\left[\frac{\partial^2 p}{\partial x_i \partial t}\right] = (-G \bar{\xi} + \frac{\delta \xi}{\delta t}) \nu_i - G g^{\alpha\beta} \xi_{,\alpha} x_{i,\beta} \qquad (5.35)$$

$$[\rho_{,ij}] = \bar{\mathcal{S}} \nu_i \nu_j + g^{\alpha\beta} \mathcal{S}_{,\alpha} (\nu_i x_{j,\beta} + \nu_j x_{i,\beta})$$

$$- \mathcal{S} g^{\alpha\beta} g^{\sigma\tau} b_{\alpha\sigma} x_{i,\beta} x_{j,\tau} \qquad (5.36)$$

$$\left[\frac{\partial^2 \rho}{\partial x_i \partial t}\right] = (\theta \bar{\mathcal{S}} + \frac{\delta \mathcal{Y}}{\delta t}) \nu_i - \theta g^{\alpha\beta} \mathcal{S}_{,\alpha}\, x_{i,\beta} \qquad (5.37)$$

$$[F_i, jk] = \bar{\eta}_i\, \nu_j\, \nu_k + g^{\alpha\beta}\, \eta_{i,\alpha}\, (\nu_j\, x_{k,\beta} + \nu_k\, x_{j,\beta})$$

$$- \eta_i\, g^{\alpha\beta}\, g^{\sigma\gamma}\, b_{\alpha\sigma}\, x_{j,\beta}\, x_{k,\gamma} \qquad (5.38)$$

The third order compatibility condition for the quantity p as given by (3) is,

$$[p, ijk] = \bar{\xi}\, \nu_i\, \nu_k + \xi_{,\alpha}\, g^{\alpha\beta}\, (\nu_j\, \nu_k\, x_{i,\beta} + \nu_i\, \nu_j\, x_{k,\beta})$$

$$- \xi\, b_{\alpha\sigma}\, g^{\alpha\beta}\, g^{\gamma\sigma}\, (\nu_i\, x_{j,\beta}\, x_{k,\gamma} + \nu_j\, x_{i,\beta}\, x_{k,\gamma} + \nu_k\, x_{i,\beta}\, x_{j,\gamma}) \qquad (5.39)$$

Where the quantites λ_i, $\bar{\xi}$, $\bar{\mathcal{S}}$ and $\bar{\eta}_i$ are new fuctions defined on the surface $\Sigma(t)$ and $b_{\alpha\sigma}$ are the components of the second fundamental form of the surface .The relations (5.32),(5.34),(5.36),(5.38), and (5.39) are called geometrical conditions of compatibility and the equation (5.33),(5.35) and (5.37) are called the kinematical components of compatibility . Since $x_{i,\beta}$ are the components of the vectors tangential to the surface $\Sigma(t)$, we have,

$$\lambda_i\, x_{i,\beta} = \lambda \nu_i\, x_{i,\beta} = 0 \qquad (5.40)$$

and

$$\lambda_{i,\alpha}\, x_{i,\beta} = \lambda_{,\alpha} \nu_{i,\beta} + \lambda \nu_{i,\alpha}\, x_{i,\beta}$$

$$= (\lambda \nu_i)_{,\alpha} x_{i,\beta} = -\lambda \nu_i \; x_{i,\alpha\beta} \qquad (5.41)$$

$$= -\lambda \, \nu_i \, \nu_i \, b_{\alpha\beta} = -\lambda b_{\alpha\beta}$$

where $x_{i,\alpha\beta}$ are the components of the second covariant derivative based on the metric of the surface $\Sigma(t)$ .Contracting the indices i abd j in (5.32) and using (5.40) and (5.41) we get .

$$[U_{i,ik}] = (\bar{\lambda}_i \, \nu_i) \, \nu_k + g^{\alpha\beta} \, \lambda_{i,\alpha} \nu_i \; x_{k,\beta} - 2 \lambda \Omega \, \nu_k \qquad (5.42)$$

where Ω is the mean curvature of the surface $\Sigma(t)$. But since,

$$\lambda_{i,\alpha} \, \nu_i = (\lambda_i \nu_i)_{,\alpha} - \lambda_i \, \nu_{i,\alpha}$$

$$= \lambda_{,\alpha} - \lambda \nu_i \; \nu_{i,\alpha} = \lambda_{,\alpha} \qquad (5.43)$$

the equation (5.42) becomes.

$$[U_{i,ik}] = (\bar{\lambda}_i \nu_i - 2 \lambda \Omega) \, \nu_k + g^{\alpha\beta} \; \lambda_{,\alpha} \, x_{k,\beta} \qquad (5.44)$$

Now multiplying (5.44) by ν_k we get

$$[U_{i,ik}]\nu_k = (\bar{\lambda}_i \, \nu_i - 2 \lambda \Omega) \qquad (5.45)$$

Similary

$$[F_{i,ik}]\nu_k = \bar{\eta}_i \, \nu_i - 2 \eta \Omega \qquad (5.46)$$

If P_1 and P_2 be the values of a quantity P on sides 1 and 2 respectively of the surface $\Sigma(t)$, the discontinuity in the product PQ is given by

$$[PQ] = Q_2 [P] + P_2 [Q] - [P] [Q]$$

If the gas is at rest on the side 2 of $\Sigma(t)$ and if the pressure and density are constant on this side of the surface as in section 5, then

$$[PQ] = -[P] [Q] \tag{5.47}$$

provided the quantities P and Q involve derivatives of the pressure p or density ρ as a factor or have, as a factor, the velocity components u_i or their derivaties. Thus, we have

$$\left[\frac{\partial^2 u_i}{\partial x_j \partial t}\right] \nu_j = -G \bar{\lambda}_i + \frac{\partial \lambda_i}{\partial t} \tag{5.48}$$

$$[p_{,ij}] \nu_j = \bar{\xi} \nu_i + g^{\alpha\beta} \xi_{,\alpha} x^i_{,\beta} \tag{5.49}$$

$$\left[\frac{\partial^2 p}{\partial x_i \partial t}\right] \nu_i = -G \bar{\xi} + \frac{\partial \xi}{\partial t} \tag{5.50}$$

$$\left[\frac{\partial^2 \rho}{\partial x_i \partial t}\right] \nu_i = G \bar{\zeta} + \frac{\partial \zeta}{\partial t} \tag{5.51}$$

$$[p_{,ijk}] \nu_j \nu_k = \bar{\xi} \nu_i \; g^{\alpha\beta} \xi_{,\alpha} x^i_{,\beta} \tag{5.52}$$

And with the help of (5.47) we have

$$[p_{,i} \; U_{i,j}] \, \nu_j = [p_{,i} \, U_{i,j}] \nu_j = \gamma\lambda$$

$$[p_{,j} \; U_{i,j}] \, \nu_j = -\gamma\lambda$$

$$[U_{i,j} \; U_{j,k}] \, \nu_i \, \nu_j = -\lambda^2 \qquad (5.53)$$

$$[p_{,i} \, \frac{\partial U_i}{\partial t}] \, \nu_i \, \nu_i = G\gamma\lambda$$

$$[U_{i,j} \, \frac{\partial U_i}{\partial t}] \, \nu_j = G\lambda^2$$

5.APPLICATION OF THE COMPATIBILITY CONDITIONS

Differentiating the equation (5.1),(5.2),(5.3) and (5.5) with respest to x and observing that $U_i = o$ on $\Sigma(t)$,we have

$$[p_{,i} \, \frac{\partial u_i}{\partial t}] + p \, [\frac{\partial^2 u_i}{\partial x \, \partial t}] + [p_{,ij}] + \rho \, [U_{i,j} \, U_{j,k}] = o \qquad (5.54)$$

$$[\frac{\partial^2 \rho}{\partial x \, \partial t}] + [p_{,i} \; U_{i,j}] + [\rho_{,j} \, U_{i,i}] + \rho \, [U_{i,ij}] = o \qquad (5.55)$$

$$[\frac{\partial^2 p}{\partial x_i \partial t}] - \rho \, [U_{i,j} \, \frac{\partial U_i}{\partial t}] + k \, [p_{,j} U_{i,i}] + kp \, [U_{i,ij}]$$
$$+ (k-1) \, [F_{i,ij}] = o \qquad (5.56)$$

$$\rho \, [F_{i,j}] = A \, p_{,j}] \qquad (5.57)$$

Again differentiating (5.57) we get,

$[\rho_{,k} F_{i,j}] + \rho [F_{i,j,k}] = A(\rho, i, j, k)$ (5.58)

By using the Compatibility conditions of second and third order in (5.54)t to (5.58) we get

$$(\rho \frac{\partial \lambda}{\partial t} + \bar{\xi}) - \rho G \bar{\lambda}_i \nu_i = 0 \qquad (5.59)$$

$$\frac{\partial \mathcal{Y}}{\partial t} = 2\mathcal{Y}\lambda + 2\rho \lambda \Omega + G \bar{\mathcal{Y}} - \rho \bar{\lambda}_i \nu_i \qquad (5.60)$$

$$\frac{\partial \xi}{\partial t} = (k+1)\, \xi \lambda + 2\rho k \lambda \Omega + 2(k-1)\, \eta \Omega$$

$$+ G\bar{\xi} - k\rho\, \bar{\lambda}_i \nu_i - (k-1) \bar{\eta}_i \nu_i \qquad (5.61)$$

$$A\, \bar{\xi} = \rho \eta \qquad (5.62)$$

$$\mathcal{Y} \eta + A\, \bar{\bar{\xi}} = \rho \eta_i \nu_i \qquad (5.63)$$

Substituting for kp in (5.61) from (5.27) we obtain,

$$\frac{d\xi}{dt} = \frac{(k+1)}{2} \xi \lambda + \rho G \lambda \Omega + \frac{(k-1)}{2\lambda}$$

$$(\eta \bar{\lambda}_i \nu_i - \lambda \bar{\eta}_i \nu_i) \qquad (5.64)$$

from (5.59) and (5.61) we have,

$$\rho\left(\frac{\partial\lambda}{\partial t} + \frac{\eta}{A}\right) = \rho G\,\bar{\lambda}\; iV_1 \qquad (5.65)$$

Again from (5.62) and (5.63) we have.

$$G\rho\,\bar{\eta}_1\,\nu_1 = \rho\eta\,(\lambda + B) \qquad (5.66)$$

with the help of (5.64),(5.65) and (5.66) we get,

$$\frac{\partial\xi}{\partial t} = \frac{(k+1)}{2}\,\xi\lambda + \rho G^2\lambda\,\Omega$$

$$+\frac{(k-1)}{2\lambda}\left\{\frac{\eta}{B}\left(\frac{\partial\lambda}{\partial t} + \frac{\eta}{A}\right) - \frac{\lambda\eta}{B}\,(\lambda + B)\right\} \qquad (5.67)$$

Differentiating the equation (5.28)-(5.30) with respect to time we have,

$$\frac{\partial\xi}{\partial t} = \rho G\,\frac{\partial\lambda}{\partial t}\,, \qquad \frac{\partial\psi}{\partial t} = \frac{\rho}{G}\,\frac{\partial\lambda}{\partial t} \qquad (5.68)$$

Simplifying (5.67) by making use of (5.68) and (5.29) we get,

$$\frac{\partial\xi}{\partial t} = \left(\frac{(k+1)a}{2\rho G} - \frac{a}{G^2\rho}\right)\xi^2 + Ga\xi\Omega + \frac{\xi b}{B} \qquad (5.69)$$

where

$$a = \frac{G^2}{(G^2 - B)} \qquad (5.70)$$

and

$$b = \frac{2(a-1)Aa}{A} \qquad (5.71)$$

Again, with the help of (5.68),(5.28) and (5.30) we get from (5.69)

$$\frac{\partial \lambda}{\partial t} = \left(\frac{(k+1)a}{2} - \frac{a}{G^2}\right)\lambda^2 + Ga\lambda\,\Omega + \frac{\lambda b}{G} \qquad (5.72)$$

and

$$\frac{\partial \psi}{\partial t} = \left(\frac{(k+1)Ga}{\rho} - \frac{a}{G\rho}\right)\psi^2 + aG\,\psi\,\Omega + \frac{\psi b}{G} \qquad (5.73)$$

When sonic wave surface $\Sigma(t)$ are propagated into a quiescent gas, the equation (5.69),(5.72) and (5.73) give the equations for the quantities ξ , λ and ψ along the normal trajectories of these surface.From these equations one can also predict the growth and decay of the sonic discontinuities associated with the wave surface $\Sigma(t)$

Let $\Sigma(t_0)$ represent the weak discontinuity surface at the time t_0 . Then ,if σ be the distance measured from $\Sigma(t_0)$ along the normal trajectories to the family of surface $\Sigma(t)$ in the direction of propagtion $\sigma = G\,(t - t_0)$ and the quantities λ, ξ and ψ

are function of the distance σ along each of the normal trajectories Hence,

$$\frac{\partial \xi}{\partial t} = G \frac{d\xi}{d\sigma} \qquad (5.74)$$

$$\frac{\partial \lambda}{\partial t} = G \frac{d\lambda}{d\sigma} \qquad (5.75)$$

$$\frac{\partial \mathcal{Y}}{\partial t} = G \frac{d\mathcal{Y}}{d\sigma} \qquad (5.76)$$

From the equation (5.69),(5.72) and (5.73) we get,

$$\frac{d\xi}{d\sigma} = \left(\frac{(k+1)}{2G} - \frac{1}{G^3}\right) \frac{a}{G\rho}\, \xi^2 + a\xi\Omega + \frac{\xi b}{G^2} \qquad (5.77)$$

$$\frac{d\lambda}{d\sigma} = \left(\frac{(k+1)}{2G} - \frac{1}{G^3}\right) a\lambda^2 + a\lambda\Omega + \frac{\lambda b}{G^2} \qquad (5.78)$$

$$\frac{d\mathcal{Y}}{d\sigma} = \left(\frac{(k+1)}{2G} - \frac{1}{G^3}\right) \frac{aG^2}{\rho}\mathcal{Y}^2 + a\mathcal{Y}\Omega + \frac{\mathcal{Y} b}{G^2} \qquad (5.79)$$

As we shall see below , it will be convenient to use the equation (5.77),(5.78) and (5.79) in the discussions that follow.

6.PLANE AND SPHERICAL DISCONTINUITIES

In the case of plane discontinuity $\Sigma(t)$, the mean curvature Ω = o and then the equations (5.77),(5.78) and (5.79) take the

form.

$$\frac{d\xi}{d\sigma} = \left\{ \frac{(k+1)}{2G} - \frac{1}{G^3} \right\} \frac{a}{G\rho} \xi^2 + \frac{\xi b}{G^2} \tag{5.80}$$

$$\frac{d\lambda}{d\sigma} = \left\{ \frac{(k+1)}{2G} - \frac{1}{G^3} \right\} a \lambda^2 + \frac{\lambda b}{G^2} \tag{5.81}$$

and

$$\frac{d\varphi}{d\sigma} = \left\{ \frac{(k+1)}{2G} - \frac{1}{G^3} \right\} \frac{aG^2}{\rho} \varphi^2 + \frac{\varphi b}{G^2} \tag{5.82}$$

Integrating these equation we have,

$$\xi = \frac{\xi_0}{\left\{ 1 - \alpha \left(1 + \beta \frac{\xi_0}{\rho} \right) \right\}} \tag{5.83}$$

$$\lambda = \frac{\lambda_0}{\left\{ 1 - \alpha \left(1 - \beta G \lambda_0 \right) \right\}} \tag{5.84}$$

$$\varphi = \frac{\varphi_0}{\left\{ 1 - \alpha \left(1 + \beta \frac{G^2 \varphi_0}{\rho} \right) \right\}} \tag{5.85}$$

where

$$\alpha = \left(1 - \exp^{-b\sigma/G^2} \right) \tag{5.86}$$

and

$$\beta = \frac{aG}{b}\left\{\frac{(k+1)}{2G} - \frac{1}{G^{*}}\right\} \tag{5.87}$$

Σ_o , λ_0 and $\mathcal{Y}_0$ being the values of the scalars Σ_j , λ and $\mathcal{Y}$ at points of the surface $\Sigma(t_o)$ where $\sigma = o$. Substituting the values of a.b and A from the equation (5.70) ,(5.71) and (5.9) a respectively in (5.87) and observing the facts that k is always negative . Then from equation (5.83),(5.84) and (5.85) we can discuss the following result . If Σ_{jo} ,λ_0 and $\mathcal{Y}_0$ are negative the quantites Σ_j ,λ and $\mathcal{Y}$ will approach zero as the distance $\sigma \to \infty$ whereas for positive values of Σ_{jo}, λ_0 and $\mathcal{Y}_0$ these quantities become infinite for the value of σ given by.

$$\sigma = \frac{G^{*}}{b}\log\left(1+\frac{\rho}{\beta\Sigma_{jo}}\right) = \frac{G^{*}}{b}\log\left(1+\frac{1}{\beta G\lambda_0}\right)$$

$$= \frac{G^{*}}{b}\log\left(1+\frac{\rho}{\beta\, G^{*}\mathcal{Y}_0}\right) \tag{5.88}$$

It follows from the equations (5.29) that if one of the quantities Σ_{jo} , λ_0 or $\mathcal{Y}_0$ is negative or positive. Also the three ratios in (5.88) must have equal values. In the first case when the scalars are negative the weak discontinuities will decay or be damped out while in the second case when the quantities are positive,the weak discontinuities will go (until) the wave finally terminates in

a weak shock for the value of σ give in (5.88). If the weak discontinuity surface $\Sigma(t)$ consist of a family of concentric spheres the mean curvature Ω is $-1/R$ where R denotes the radii of the spheres of the family provided R is assumed to increase with time t. Replacing the distance σ by R in the equation (5.77),(5.78) and (5.79) we have,

$$\frac{d\xi}{dR} = \left\{\frac{(k+1)}{2G} - \frac{1}{G^{2}}\right\} \frac{a}{G\rho'} \xi^{2} + \xi \left\{\frac{b}{G^{2}} - \frac{a}{R}\right\} \qquad (5.89)$$

$$\frac{d\lambda}{dR} = \left\{\frac{(k+1)}{2G} - \frac{1}{G^{2}}\right\} a \lambda^{2} + \lambda \left\{\frac{b}{G^{2}} - \frac{a}{R}\right\} \qquad (5.90)$$

$$\frac{d\varphi}{dR} = \left\{\frac{(k+1)}{2G} - \frac{1}{G^{2}}\right\} \frac{aG^{2}}{\rho} \varphi^{2} + \varphi \left\{\frac{b}{G^{2}} - \frac{a}{R}\right\} \qquad (5.91)$$

integrating (5.89),(5.90) and (5.91) we get

$$\frac{1}{\xi} = \left\{\frac{1}{G^{3}} - \frac{(k+1)}{2G}\right\} \frac{ab^{a-1}\, 2\pi i}{\rho G^{2(a-1)}\sqrt{a}} R^{a} \exp^{-bR/G^{2}} \qquad (5.92)$$

$$\frac{1}{\lambda} = \left\{\frac{1}{G^{3}} - \frac{(k+1)}{2G}\right\} \frac{ab^{a-1}\, 2\pi i}{G^{2(a-1)}\sqrt{a}} R^{a} \exp^{-bR/G^{2}} \qquad (5.93)$$

and

$$\frac{1}{\varphi} = \left\{\frac{1}{G^{3}} - \frac{(k+1)}{2G}\right\} \frac{ab \quad 2\pi i}{G^{2(a-2)}\sqrt{a}} \exp^{-bR/G^{2}} \qquad (5.94)$$

where the integration has been carried out with the help of Hankel's contour . As $R \longrightarrow \infty$ ξ , λ and ψ tend to zero indicating that the weak discontinuities are damped out whereas they become indefinitely large as $R \longrightarrow 0$, showing that the weak discontinuity must degenerate into a spherical shock. This fact is borne by the Hankel's contour as well.

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CHAPTER VI

AN EXPANDING PRESSURE SHOCK IN MAGNETOTHERMAL SYSTEMS

INTRODUCTION.

If $\Sigma(t)$ to be wave surface or boundary of a disturbance propagating in a thermally coducting viscous gas. On account of the gas being viscous, the continuity in velocity and hence in density over $\Sigma(t)$ may well be assumed . One is, therefore, tempted and quite naturally too, to assume continuity in pressure as well. But since from the basic equations governing the flow it can be shown that all orders of derivatives of surface density and pressure must be continuous over the surface $\Sigma(t)$,One is led to conclude that $\Sigma(t)$ does not sustain a discontinuity of any order and as such wave of finite thickness is assumed. Since there is no mathematical or physical requirement of continuity of pressure over the surface $\Sigma(t)$, we assume a discrete discontinuity in the surface. This enables us to conceive the idea of what Thomas and Edstrom [1] called the pressure shock , which is of singnificant importance especially in the theroy of blast wave s.DL decka r[2] obtained an solution of Velocity and temperature flow behind propagating shock in a duct, our purpose in the present work is to derive the growth equation for such pressure shock in a thermally and electrically conducting viscous gas with radiations effects.

2. BOUNDARY CONDITIONS

The boundary conditions appropriate to the problem under

consideration are,

$$[U_i] = 0 \quad , \quad U_i = 0 \tag{6.1}$$

$$[p_{,i}]\, n_i = 0 \tag{6.2}$$

$$\left[\frac{dT}{dt}\right] = 0 \tag{6.3}$$

$$\varepsilon^{ijk} H_j J_k = 0 \tag{6.4}$$

where the symbols have their usual meaning . Equation (6.1) expresses the condition of continuity of velocity over $\Sigma(t)$,while the condition given by equation (6.2),stating that the normal directional derivative of the pressure vanishes on the flow side of $\Sigma(t)$, is suggested by usual pressure condition in boundary layer theory . This condition is applicable since the flow in the immediate neighbourhood of $\Sigma(t)$ and the flow surrounding a moving body in a viscous fluid are similar.The equation (6.3) which involves the total time derivative of T, expressess the condition that a material particle has a stationary temperature at the time of its contact with the rear of the surface $\Sigma(t)$. As the velocity vanishes on the surface $\Sigma(t)$ this condition can also be expressed as,

$$\frac{dT}{dt} = 0 \tag{6.5}$$

The condition (6.4) expresses the fact that just behind the shock surface the Lorentz foce is zero.[3]

3. JUMP CONDITIONS

In their recent paper Verma & Srivastava [4] have shown that in radiation gas dynamics the internal energy is equal to the sum of the internal energy of ordinary gas and radiation energy while the total pressure is the ordinary gas pressure and radiation pressure. Then the shock conditions as given by Pant & Mishra [6] after modifying for the case of radiation magnetogasdynamics, become,

$$[H_n] = 0 \tag{6.6}$$

$$[\rho (U_n - G)] = 0 \tag{6.7}$$

$$H_n [U_n] - [(U_n - G) H] + \mu [H_{i,j}] n_j = 0 \tag{6.8}$$

$$\rho_i (U_{in} - G) [U_i] - [\sigma_{ij}] n_j - \frac{1}{4\pi} (H_n [H_i] - \frac{1}{2} [H^2]) = 0 \tag{6.9}$$

$$\rho_i (U_{in} - G) [\tfrac{1}{2} U^2 + E] - [\sigma_{ij} (U_i - G n_i)] n_j - k [T_{,i}] n_i$$

$$+ \frac{\mu}{4\pi} [H_j (H_{i,j} - H_{j,i})] n_i - \frac{1}{4\pi} \left\{ ([\mu^{\alpha} H_{\alpha}]) - [U_n - G] \frac{H^2}{2} \right\} + F = 0 \tag{6.10}$$

where the braket [ ] stands for the jump across the shock surface $\Sigma(t)$ of the quantity enclosed. Let us assume that the unit normal n to the surface $\Sigma(t)$ having components n_i is directed into the region of wave propagtion so that the normal velocity G of the surface $\Sigma(t)$ is positive. It is easy to observe the following relations,

$$U_n = U_i n_i \quad , H_n = H_i n_i \tag{6.11}$$

and

$$u = \frac{c^2}{4\pi \sigma}$$

$$E = \frac{p}{\rho(\gamma-1)} + \frac{aT^4}{\rho} \tag{6.12}$$

$$\sigma_{ij} = -\left(p + \frac{aT^4}{3}\right)\delta_{ij} + \eta(U_{i,j}+U_{j,i}) + \left(\zeta - \frac{2}{3}\eta\right) U_{l,l}\delta_{ij}$$

and

$$F_i = - D\,(aT^4)_{,i} \tag{6.13}$$

where D is the diffusion coefficient defined by $D = C/3\chi\rho$ and C being opacity and velocity of light respectively . Also , K is the coefficent of thermal conductivity, H is magnetic field strength with components H_i , σ is the electrical conductivity , $aT^4/3$ and aT^4/ρ are the radiation pressure and radiation energy respectively . η and ζ are the two coefficients of viscosity and F_i is the radiation flux . We have assumed that jump in radiation flux is given by,

$$[F_i n_i] = F$$

Let us consider the jump in a function f and its derivatives across the surface $\Sigma(t)$. If we suppose that

$$[f] = A \tag{6.14}$$

and

$$[f_{,i}]n_i = B \qquad (6.15)$$

then, from the well known compatibility conditions given by Thomas [5], we have

$$[f_{,i}] = B\, n_i + g^{\alpha\beta} A_{,\alpha} x^i_{,\beta} \qquad (6.17)$$

$$\left[\frac{\partial f}{\partial t}\right] = -GB + \frac{\delta A}{\delta t} \qquad (6.18)$$

wher the operator $\frac{\delta}{\delta t}$ is defined as

$$\frac{\delta z}{\delta t} = \frac{\partial z}{\partial t} + z_{,i}\, G n_i \qquad (6.19)$$

On account of the continuity of the velocity and assumed continuity of the inensity of the magnetic field over $\Sigma(t)$, the compatibility conditions of the first order for these quantities are (Thomas [6])

$$[U_{i,j}] = \theta_i \qquad (6.20)$$

and

$$[H_{i,j}]n_j = \phi_i \qquad (6.21)$$

where θ_i and ϕ_i functions defined over the surface $\Sigma(t)$. In view of the relations (6.17), (6.20) and the boundary conditions (6.1) to (6.4), the shock conditions (6.8) to (6.10) become.

$$G[H_i] - \mu\phi_i = 0 \qquad (6.22)$$

$$[(p + \frac{aT^4}{3})n_i + \frac{[H^2]}{8\pi} n_i - (\frac{1}{3}\eta + y)\theta_i n) n_i$$

$$-\eta\theta_i - \frac{Hn}{4\pi}[H_i] = 0 \qquad (6.23)$$

$$-\frac{\rho_1[GRET]}{(\tau-1)} + [aT^4]G - K[T_{,i}]n_i - \frac{G[H^2]}{8\pi} + F = 0 \qquad (6.24)$$

Let the parametric equation to the surface $\Sigma(t)$ be given by $x^i = x^i(U^1, U^2, t)$ where U^α $(\alpha = 1,2)$ are the parametric coordinates. The quantities x^i_α then denote the derivatives of the space coordinates x^i with respect to the parametric coordinates. Let us assume that $\Sigma(t)$ is regular in the sense that the functional matrix $\| x^i_\alpha \|$ has the rank 2 at points of this surface. The quantities x^i_α for α fixed are then the components of a vector which is tangential to the surface $\Sigma(t)$. Multiplying the equation (6.23) by x^i_α and applying summation convention we find that θ_i and ϕ_i are the componentes of vector normal to the surface, so that, we my write

$$\theta_i = \theta n_i \qquad (6.25)$$

and

$$\phi_i = \phi n_i \qquad (6.26)$$

θ and ϕ being scalars defined on the surface $\Sigma(t)$. Again, from (6.23) and (6.24) we have,

$$[p + \frac{aT^4}{3}] \quad \frac{[H^2]}{8\pi} + (\frac{4}{3}\eta + \gamma)\,\theta_i\, n_i \qquad (6.27)$$

and

$$K\,[T,i]\, n_i = -\theta\,\{\frac{\rho R[T]}{(\gamma-1)} + \frac{[H^2]}{8\pi} + [aT^4] - \frac{F}{\theta}\} \qquad (6.28)$$

4. EXPAUDING GROWTH EQUATION

As a consequence of the usual relation,

$$p = \rho RT \qquad \text{(R is the gas constant)} \qquad (6.29)$$

and (6.27) we get,

$$[T] = \frac{(4/3\eta + \gamma)\lambda}{R\rho} \qquad (6.30)$$

where

$$\lambda = \theta_i\, n_i - \frac{[H^2]}{8\pi(4/3\eta+\gamma)} - \frac{[aT^4]}{3(4/3\eta+\gamma)} \qquad (6.31)$$

From (6.30) and (6.28) we obtain

$$K[T,i]\, n_i = -\theta\,\{\frac{\lambda(4/3\eta+\gamma)}{(\gamma-1)} + \frac{[H^2]}{8\pi} + [aT^4] - \frac{F}{\theta}\} \qquad (6.32)$$

Using the equation (6.18) we get

$$\left[\frac{\partial t}{\partial t}\right] = -G\,[T,i] + \frac{\delta[T]}{\delta t} \qquad (6.33)$$

which in view of the boundary condition (6.9) and the relations (6.30) and (6.32) gives

$$\frac{\delta}{\delta t}\frac{(4/3\eta+y)}{R\rho} + \frac{G^2}{k}\frac{\lambda(4/3\eta+y)}{(\gamma-1)} + \frac{[H^2]}{8\pi}$$

$$+ [aT^4] - \frac{F}{G} = 0 \qquad (6.34)$$

or

$$\frac{k}{R\rho}\frac{\delta\lambda}{\delta t} + \frac{G^2\lambda}{(\gamma-1)} + \frac{G^2[H^2]}{8\pi(4/3\eta+y)} + \frac{G^2[aT^4]}{(4/3\,\eta+y)}$$

$$-\frac{FG}{(4/3\,\eta+y)} = 0 \qquad (6.35)$$

which is the growth equation in radiation magnetogasdynamics.

Applying (6.17) and (6.18) in the equation of continuity,

$$\frac{\partial\rho}{\partial t} + \rho_{,i}\,U_i + \rho\,U_{,i} = 0 \qquad (6.36)$$

we obtain,

$$-G\,[\rho_{,i}]\,n_i + \rho\,\theta_i\,n_i = 0 \qquad (6.37)$$

Differentiating (6.29) with respect to x^i and evaluating jump of quantities on both sides we get

$$[p,_i] = \rho R [T,_i] + R [\rho,_i][T] + T_1 [\rho,_i] R \qquad (6.38)$$

where T_1 is the constant temperature in the uniform region infront of the shock . Multiplying (6.38) by n_i , summing with respect to the index i, and using the boundary condition (6.2) ,we find that the growth equation (6.35) can be written in the from

$$\lambda(4/3\, \eta + y) \left\{ \frac{K\lambda}{R\rho} - \frac{G^2}{(\gamma-1)} + \frac{KT_1}{(4/3\, \eta + y)} \right\}$$

$$+ \frac{[H^2]}{8\pi} \left\{ \frac{K\lambda}{R\rho} - G^2 + \frac{KT_1}{(4/3\, \eta + y)} \right\}$$

$$+ \frac{aT^4}{3} \left\{ \frac{K\lambda}{R\rho} - 3G^2 + \frac{KT_1}{(4/3\, \eta + y)} \right\} + F\,G = 0 \qquad (6.39)$$

It is easy to see that in the absence of magnetic field and radiation effects the equation (6.39) reduces to the equation obtained by Thomas and Edstrom [1] .

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CHAPTER VII

Differential effects shocks with Heat addition in a Conducting Region

1. INTRODUCTION :-

The differential effects of shock waves in fluids have been discussed by many authors , viz. Kanwal [1],Truesdell [2],Hayes [3] etc. to mention only a few. Pai [4] discussed shock wave with heat addition but could not rigorously modify the equations governing the flow to take it into account . Mishra and Verma [5] and Verma [6] obtained the differential effects of shocks with heat addition in non-conducting gases. Recently Khare,Upadhyay and Mata Amber [7] introduced a new heat addition vector (which they called as HKM vector) to take heat addition vector into account in proper manner . The decay of saw Tooth profiles in a chemically reacting gases was elaborated by shukla and Singh [8]. In this chapter , It has been discussed the differential effect of shock with heat addition in electrically conducting gases by considering the same vector for heat addition . We have derived jump conditions across the shock by integrating the equations governing the flow and field . We have also obtained the density strength of the shock,expressions for the vorticity jump , derivatives of velocity , pressure , density and manganetic field and derived the well known particular cases.

2. BASIC EQUATION

Let the shock surface $\Sigma(t)$ in three dimensional unsteady flow be represented by continuously differentiable functions

$$X_i = X_i\,(y^\alpha, t) \qquad (7.1)$$

where X_i are the rectangular co-ordinates of a point on the shock surface and y^α are the Gaussian co-ordintes of the point . As usual, the range of Latin indices, referring to spatial tensor is 1,2,3 and that of greek indices to referring to surface tensor is I,II . Assuming the fluid to be infinite electrical conductivity, the flow and field equations in case of heat addition are,

$$\frac{\partial \rho}{\partial t} + \rho_{,i}\, U_i + \rho U_{i,i} = o \qquad (7.2)$$

$$\rho \frac{\partial U_i}{\partial t} + \rho U_j\, U_{i,j} + p_{,i} + \frac{1}{4\pi} H_k H_{k,i} - \frac{1}{4\pi} H_{i,j} H_j = o \qquad (7.3)$$

$$H_{i,i} = o \qquad (7.4)$$

$$\frac{\partial H_i}{\partial t} - U_{i,j} H_j + H_{i,j} U_j + H_i U_{k,k} = 0 \qquad (7.5)$$

$$\frac{\partial}{\partial t}\left(\tfrac{1}{2}\rho U^2 + \rho e + \frac{H^2}{8\pi}\right) + \left(U_i\left(\tfrac{1}{2}\rho U^2 + \rho e + p + \frac{H^2}{4\pi}\right)\right)_{,i} - \tfrac{1}{4}\pi\,(U_j H_j H_i)_{,i} + (a\, F_i)_{,i} = o \qquad (7.6)$$

where p , ρ , U_i , e and H_i stand for pressur , density , components of velocity, internal energy and field component respectively ; a is a constant and F_i is the heat addition vector

as defined by Khare , Upadhaya and Mata Amber [73].

Integrating the equations (7.2) to (7.6) across the shock we get,

$$[\rho V_n] = 0 \tag{7.7}$$

$$\rho_1 V_{1n} [V_i] + [p^*] n_i - \frac{1}{4\pi} H_{1n} [H_i] = 0 \tag{7.8}$$

$$H_n = H_{1n} \tag{7.9}$$

$$[H_i V_n] - H_{1n} [V_i] = 0 \tag{7.10}$$

$$\rho_1 V_{1n} \left[\frac{V^2}{2} + e + \frac{p}{\rho} + \frac{H^2}{4\pi\rho} \right] - [H_i V_i] \frac{H_{1n}}{4\pi}$$

$$+ a[F_n] = 0 \tag{7.11}$$

where $p^* = p + H^2/8\pi$, V_i are the components of the velocity of the fluid relative to the shock that is $V_i = U_i - Gn_i$, G being the speed of the shock along its normal.

3. STRENGTH OF THE SHOCK

Let δ, the density strength of the shock be defined as

$$\delta = \frac{[\rho]}{\rho_1}$$

so that the ,equations (7.7) to (7.10) give

$$[V_i] = \frac{\delta_{in} H_{it} H_{in} l_i}{4\pi \rho_1 V_{in}^2 - (1+\delta) H_{in}^2} - \frac{\delta}{1+\delta} V_{in} n_i \qquad (7.12)$$

$$[H_i] = \frac{4\pi \ \delta \ \rho_1 V_{in}^2 H_{it}}{4\pi \rho_1 V_{in}^2 - (1+\delta) H_{in}^2} \ l_i \qquad (7.13)$$

$$[p^*] = \frac{\delta}{1+\delta} \rho_1 V_{in}^2 \qquad (7.14)$$

where l_i denote the components of the tangent to the shock and $H_t = H_i \ l_i$.

It is also possible to obtain all the differential effect of shock waves in mangnetogasdynamics with heat addition by applying techniques followed by various authors for corresponding problem in ordinary gases . We here confine ourselves only to the unsteady plane shock waves and assume that the mangnetic field has components (0,0,H) so that the energy equation becomes

$$\frac{\partial p}{\partial t} + \frac{H}{4\pi} + \frac{\partial H}{\partial t} - \rho U_i \frac{\partial U_i}{\partial t} - \rho U_i U_j (U_{i,j} + \frac{H^2}{4\pi} + \gamma p) U_{k,k}$$

$$+ \beta F_{i,i} = 0 \qquad (7.15)$$

where β is another constant . The effective sound speed S in an electrically conducting medium is then given by

$$S^2 = \frac{\gamma p}{\rho} + \frac{H^2}{4\pi\rho}$$

So that the equation (7.15) can be written as

$$\frac{\partial p^*}{\partial t} - \rho U_i \frac{\partial U_i}{\partial t} - \rho U_i U_j U_{i,j} + S^2 \rho U_{k,k} + \delta F_{i,i} = 0 \quad (7.16)$$

The jump conditions (7.12) to (7.14) then take the form,

$$[V_i] = \frac{-\delta}{1+\delta} V_{in} n_i \quad (7.17)$$

$$[H] = \delta H_1 \quad (7.18)$$

$$[p^*] = \frac{\delta}{1+\delta} \rho_1 V_{in}^2 \quad (7.19)$$

and the energy balance across the shock gives.

$$\left[\frac{V_n^2}{2} + \frac{p}{\rho(\tau-1)} + \frac{H^2}{4\pi\rho}\right] + \frac{\alpha}{\rho_1 V_{in}} [F_n] = 0 \quad (7.20)$$

With the help of (7.17) to (7.20) we have,

$$(\tau-2) \frac{H_1^2}{4\pi\rho_1} \delta^2 + \left\{(\tau-4) \frac{H_1^2}{4\pi\rho_1} - \frac{2\tau p_1}{\rho_1} - (\tau-1) V_{in}^2\right\} \delta$$

$$+ 2\left(V_{in}^2 - \frac{\tau p_1}{\rho_1} - \frac{H_1^2}{4\pi\rho_1}\right) + \frac{2(1+\delta)^2 \alpha(\tau-1) [F_n]}{\delta \rho_1 V_{in}} = 0$$

Hence,

$$B_1^2 = \frac{\tau-2}{2} \delta \frac{H_1^2}{4\pi\rho_1} - \frac{V_{in}^2}{2(1+\delta)} \{(\tau-1) \delta-2\}$$

$$+ \frac{\alpha(\gamma-1)\,(1+\delta)\,[F_n]}{\delta \rho_1 \, V_{1n}} \qquad (7.21)$$

which shows that there is no effect of heat addition on the strength of the shock if the normal component of heat addition vector is continuous across the shock . It is easy to see that when H->o the equations (7.21) reduces to the case of Khare et al. [7] which, for ordinary gases is the general form of Verma [6].

4. JUMPS IN VORTICITY, GRADIENTS OF VELOCITY, DENSITY AND MANGNETIC FIELD

The vorticity W is given by,

$$W\, e_{ij} = U_{j,i} - U_{i,j}$$

where , $e_{ij}\, l_j = - n_i$ and $e_{12} = - e_{21} = 1$.

Following Kanwal [1] , we have,

$$W = \frac{1}{V_N}\left(A_i\, V_i + \frac{B}{\rho} + \frac{\delta}{\delta t}\, V_i\, l_i\right) \qquad (7.22)$$

where

$$U_{i,j}\, l_i = A_i \quad , \quad P^*_{,j}\, l_i = B \qquad (7.23)$$

and

$$\frac{\delta}{\delta t}(\;) = \frac{\partial}{\partial t}[\;] + G\, n_i\, [\;]_{,i}$$

To evaluate the gradients of velocity components we get ,

$$\begin{Vmatrix} C_{11} & C_{21} \\ & \\ C_{12} & C_{22} \end{Vmatrix} = \begin{Vmatrix} 1_1 & 1_2 \\ & \\ V_1 & V_2 \end{Vmatrix}$$

and

$$\begin{Vmatrix} D_{11} & D_{21} \\ D_{12} & D_{22} \end{Vmatrix} = \begin{Vmatrix} V_2 V_n & n_1 V_n \\ -V_1 V_n & n_2 V_n \end{Vmatrix}$$

so that

$D_{ik} C_{kj} = \delta_{ij}$ and $D_{k,i} C_{jk} = \delta_{ij}$

Let the quantities B_{ij} be defined as

$$B_{ij} = U_{i;m} C_{ik} C_{mj} \qquad (7.26)$$

Then the energy equation (7.16) in the present notation becomes,

$$\rho V_i V_j V_{i,j} = \frac{1}{\rho} \frac{\delta p^*}{\delta t} - V_i \frac{\delta}{\delta t} V_i$$

$$+ S^* V_{i,i} + \frac{\beta}{\rho} F_{i,i}$$

From the equation (7.23) and (7.24) we have,

$$\left\| \begin{matrix} B_{11} & B_{12} \\ \\ B_{21} & B_{22} \end{matrix} \right\| = \left\| \begin{matrix} A_i \; l_i; - (\frac{B}{\rho} + \frac{\delta V_i}{\delta t} \, l_i) \\ \\ A_i \; V_i; (\frac{1}{\rho} \frac{\delta p^*}{\delta t} - \frac{\delta V_k}{\delta t} V_k \, SV_k, k + \frac{B}{\rho} F_{i,i}) \end{matrix} \right\|$$

From (7.25) we observe that B_{11}, B_{12} and B_{21} are given in terms of the known quantities upstreams of the shock while

$$B_{22} = \frac{\rho \, V^*_n (\frac{1}{\rho} \frac{\delta p^*}{\delta t} - \frac{\delta V_k}{\delta t} V_k) + \rho S^* (B_{11} V^* - (B_{12} + B_{21}) V_t) + B V_n^2 F_{i,i}}{\rho \, (V_n^2 - B^*)}$$

where $V_t = V_i \; l_i$

As in Kanwal [1] the jumps in gradients of density and magnetic field are given by

$$\rho , j = d_i \; D_{ij} \; ; \; H , j = \frac{H_i}{\rho_i} d_i \; D_{ij}$$

where $\rho_i \; \delta' = C = d_1$ and $d_2 = \frac{-\delta \rho}{\delta t} - \rho V_k , k$

a prime denoting differentiation with respect to the arc length which increases algebraically in the direction of the unit tangent vector.

5. THE QUANTITIES B_{ij} AND d_i

These quantities can be evaluated with the help of the geometrical formulas

$U_i = K n_i$, $n'_i = -K l_i$.

where K is curvature of the shock . We have,

$$\frac{\delta n_i}{\delta t} = - G' \, l_i \qquad (7.26)$$

Thus $V'_{1n} = (U'_{1i} \, n'_i - G') - (K u_t + G')$

and

$$\frac{\delta V_{1n}}{\delta t} = [U_{1i} \frac{\delta_{ni}}{\delta t} - \frac{\delta G}{\delta t}] = -(u'_t G' + \frac{\delta G}{\delta t})$$

where we have used the fact that $u_{1t} = u_t$.

We also have , from the relation prior to (7.21)

$$\delta' = - 2D \; V_{1n} \, [K u_t + G'] \qquad (7.27)$$

$$\frac{\delta \rho}{\delta t} = - 2D \, \rho_1 \, V_{1n} \, [u_t \, G' + \frac{\delta G}{\delta t}] \qquad (7.28)$$

where,

$$D = \frac{(\tau - 1)\,\delta - 2}{2\delta(\tau - 2)\dfrac{H_1^2}{4\pi\rho_1} - (\tau - 1)\, V_{1n}^2 + \dfrac{2\alpha}{\rho_1}(\tau - 1)\,[F_n](\delta^2 - 1)} \qquad (7.29)$$

It can be verified that if the normal component of heat addition vector is continuous over the shock surface the problem is reduced to magnetogasdynamic case. The values of A_1, B, C,

$\frac{\delta u_1}{\delta t}$, $\frac{\delta H}{\delta t}$, $\frac{\delta p^*}{\delta t}$, B_1, and d_1 for the above value of D can be obtained as in Kanwal (1) Substituting for A_1, B and $\frac{\delta u_1}{\delta t}$ in (7.22) we have

$$w = \frac{(Ku_t + G')\delta^2}{1 + \delta}$$

which has the same form as in ordinary gas dynamics except that here it is also a function of the magnetic field and heat addition vector.

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